The Geometric Supposer:
Triangles

Apple

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CENTER FOR LEARNING TECHNOLOGY
EDUCATION DEVELOPMENT CENTER, INC.

SOFTWARE DESIGN: Judah L. Schwartz and Michal Yerushalmy

MANUAL DEVELOPMENT: Judah L. Schwartz, Michal Yerushalmy and Myles Gordon

For information or a free microcomputer courseware catalog, write or call:

Sunburst Communications, Inc.
39 Washington Avenue
Pleasantville, NY 10570

In the United States call toll-free 800-431-1934
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This is THE GEOMETRIC SUPPOSER: Triangles, part of the mathematics software series of Education Development Center, Inc. THE GEOMETRIC SUPPOSER: Points and Lines, programs similar in concept to the Triangles program, will be published by SUNBURST COMMUNICATIONS later this year.

About the designers:

JUDAH L. SCHWARTZ is Professor of Engineering Science and Education at Massachusetts Institute of Technology, Senior Developer for the Center for Learning Technology at EDC, and Visiting Professor at the Harvard Graduate School of Education. At Harvard, he is also Co-Director of the NIE-funded Educational Technology Center, a five-year, $7.7 million effort to find new ways of using technology in science, mathematics, and computer science education. He developed SemCalc and is primary developer of the mathematics software series of the Center for Learning Technology, published through Sunburst Communications, Inc. In the past few years, Dr. Schwartz has become widely known as a writer, a speaker, and for his thoughtful approach to computers and learning. His research interests include cognitive development in the learning of mathematics and problem-solving, uses of computers to augment human intuition, and the process and substance of undergraduate and continuing education. Previously, as Co-director of the Cognitive Research Group at EDC, he conducted several research and demonstration projects designed to answer questions about how people learn to solve problems, and to investigate the use of microcomputers and printed materials to teach an approach to problem-solving. Dr. Schwartz holds an M.S. in Physics from Columbia University and a Ph.D. in Physics from New York University.

MICHAL YERUSHALMY is a Research Associate at the Center for Learning Technology at EDC. At EDC, she has worked closely with Dr. Schwartz on the mathematics software series. Her primary research interests are mathematics learning and teaching. She has conducted studies on new approaches to teaching mathematics and implementing specially designed software. Ms. Yerushalmy has over ten years of experience as a high school mathematics and computer science teacher and has served as a high school mathematics department chairman. In addition, she has taught procedural thinking and problem-solving to children in grades four through seven, using a special curriculum that introduces mathematical and logical concepts through graphics and games programming. Ms. Yerushalmy has B.S. degrees in Mathematics and Education and an M.S. degree in Education (the Department of Teaching Technology and Sciences) from the Technion in Haifa, Israel. She is currently a doctoral student at the Harvard Graduate School of Education.

The CENTER FOR LEARNING TECHNOLOGY was established in 1982 by Education Development Center, which for over twenty-five years has been a pioneer in the use of new technologies as tools for teachers and learners. In addition to software development, the Center's activities include research, policy analysis, and videotape and videodisc production.

The Center's mathematics software series, published by Sunburst Communications, Inc., addresses particularly troublesome areas of the mathematics curriculum at the elementary, middle, and high school levels. In addition to The Geometric Supposer, programs in the series include Number Quest, which introduces search strategies with whole

numbers, fractions, ordered number pairs, and ordered number triples; Word Quest, which explores the use of a mathematical problem-solving strategy in a non-mathematical context, building vocabulary and dictionary skills; Power Drill, which develops estimation skills in addition, subtraction, multiplication, and division; Get to the Point, which focuses on order of magnitude (i.e., where does the decimal point go?) in computation with decimals; and SemCalc, a tool for solving word problems.

The Center for Learning Technology is also developing software for writing instruction, programs in early reading instruction that use speech synthesis and recognition technologies, and a college level program in behavioral psychology that employs interactive videodisc. The Center for Learning Technology is a member of the Harvard-based, NIE-funded Educational Technology Center consortium.

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INTRODUCTION

There is something odd about the way we teach mathematics in our schools. We make little or no provision for students to play an active and generative role in learning mathematics and we teach mathematics as if we expected that students will never have occasion to invent new mathematics.

We don't teach language that way. If we did, we would never require students to write an original piece of prose or poetry. We would simply require them to recognize, appreciate, and memorize the great pieces of language of the past, literary equivalents of the Pythagorean Theorem and the Law of Cosines.

Most mathematics instruction is a kind of satire on the nature of mathematical thinking and the process of creating new mathematics. When a teacher assigns a theorem to prove, students assume that the theorem is true and that a proof can be found. The central activity of creating new mathematics--the making and the testing of conjectures--is absent from the classroom.

Making conjectures in geometry requires exploring the relationships that do or do not hold among geometrical "objects." And this exploration is made much easier if one can construct and manipulate these "objects." In the normal course of events, however, it is not feasible to ask students (or anyone else) to do a great deal of construction. Accurate constructions are difficult to make, and because of that difficulty, it is not reasonable to expect people to repeat constructions over and over again in order to generate a repertoire of cases on which to base a conjecture.

THE GEOMETRIC SUPPOSER is designed to overcome these obstacles and thereby help the student to become a potent and nimble conjecture-maker. It allows students to make any construction they wish on any triangle. The program records that construction as a procedure that can then be executed with any other triangle. As a result, the user can explore whether the consequences of a given construction on a given triangle are dependent on some particular property of that triangle, or if the result can be generalized.

Needless to say, neither possibility nor plausibility constitutes proof. Proof remains critical to both the creating and the learning of mathematics. But with the aid of THE GEOMETRIC SUPPOSER as intellectual amplifier, conjecture can assume its proper role as a key activity in the learning and teaching of geometry.

The fact that the SUPPOSER makes it possible to make constructions easily and quickly does not mean that the student should never be asked to use a straightedge and compass to bisect angles or to erect perpendiculars. Making constructions with tangible straightedges and compasses is essential. Direct experience with these tools makes the power of the SUPPOSER apparent.

What is THE GEOMETRIC SUPPOSER?

THE GEOMETRIC SUPPOSER is a microcomputer program that allows the user to carry out with ease constructions that are possible using straightedge and compass. These include the construction of triangles as well as the drawing of segments, medians, altitudes, parallels, perpendiculars, perpendicular bisectors, angle bisectors, and inscribed and circumscribed circles. In addition, the user can measure lengths, angles, areas and distances as well as arithmetic combinations of these measures, such as the sum of two angles, the product of two lengths, or the ratio of two areas.

Such a program is useful in the learning of geometry in that it allows constructions to be made accurately and easily. The real power of THE GEOMETRIC SUPPOSER, however, lies in another feature--its ability to remember and to repeat constructions. Any construction on a triangle that a user makes with the SUPPOSER may be repeated on a new triangle of the user's construction, a previously used triangle or a random right, acute, obtuse, isosceles, or equilateral triangle.

Using this feature, beginning geometry students discover that if they draw a median in a triangle, the median bisects the area of the triangle, and that this seems to be true if they repeat the construction with triangles of all shapes and sizes. Having established the plausibility of this conjecture, they can then devise a proof with conviction, growing out of some direct experience.

Students have discovered for themselves that a midsegment in a triangle is parallel to the third side of the triangle, and that the three midsegments of the triangle partition the triangle into four triangles that are congruent to one another and similar to the original triangle. They have made discoveries about the ratios of perimeters and of areas that are presented as theorems in most classrooms.

And a few students have even discovered new theorems. In so doing, all of these students are coming to understand that mathematics is a lively and open-ended enterprise, and one that, with the right tools, is accessible.

How has THE GEOMETRIC SUPPOSER been used?

The SUPPOSER was designed originally for use in high school geometry classes. At the time of this writing, however, several middle school classes, as well as a dozen or so high school classes, both public and private, are using the program. In addition, at least one group of students in a vocational-technical high school is using THE GEOMETRIC SUPPOSER.

At the middle school level, students working with the SUPPOSER are exploring geometric ideas that are not part of the standard curriculum. The high school geometry classes are using the SUPPOSER in new ways to approach the traditional content of geometry courses. Some constitute rather modest departures from usual instruction, while others are truly revolutionary in the manner in which they draw mathematics out of the students. In the vocational setting, students are working with THE GEOMETRIC SUPPOSER to augment their studies in drafting and design and to strengthen their spatial reasoning skills.

In some cases, the SUPPOSER is being used in computer laboratories. In other cases, schools are using the program in classrooms with only one microcomputer. Needless to say, the availability of hardware dictates in part how the SUPPOSER is used, but teachers have demonstrated that the SUPPOSER can be used productively regardless of hardware constraints.

(For a more detailed discussion of using THE GEOMETRIC SUPPOSER in the classroom, see Section III of this manual.)

How was THE GEOMETRIC SUPPOSER developed?

The first version of THE GEOMETRIC SUPPOSER was developed during the 1981-82 school year. The program was first tried with students in the tenth grade at Commonwealth School in Boston in October 1982. Those early trials were exciting, challenging and frustrating. Although we believed deeply that we were engaged in an important endeavor in mathematics education, we were impatient with our own inability to clarify issues and define things as crisply as we would have liked.

In the Spring of 1983, the math faculty of the Weston (Massachusetts) High School and the administration of the Weston system asked us if we would consider using Weston as a site for pursuing further development of the SUPPOSER. With the collaboration of some remarkable teachers, we implemented THE GEOMETRIC SUPPOSER on the Weston School's minicomputer.

The 1983-84 school year saw the use of the SUPPOSER by Richard Houde and two of his geometry classes at Weston. From his classes, we learned a good deal about both the content and the teaching of geometry. As a result of that experience, conceptual issues were clarified and the operation of the program ws simplified. The final

version of the SUPPOSER reflects what we have learned from students and teachers.

During the 1984-85 school year, about fifteen teachers from Boston area schools have been using the SUPPOSER in a variety of settings. As a pilot users group, they have been meeting on a monthly basis to share their experiences and exchange ideas about the use of the program in the classroom.

An Afterword

We believe that THE GEOMETRIC SUPPOSER offers the mathematics teacher a new way to approach the teaching of geometry. More broadly, we believe that students can be active participants in the learning of mathematics and even make their own mathematics, and that microcomputers can help them to do so, thereby changing the way mathematics is learned and taught at all levels.

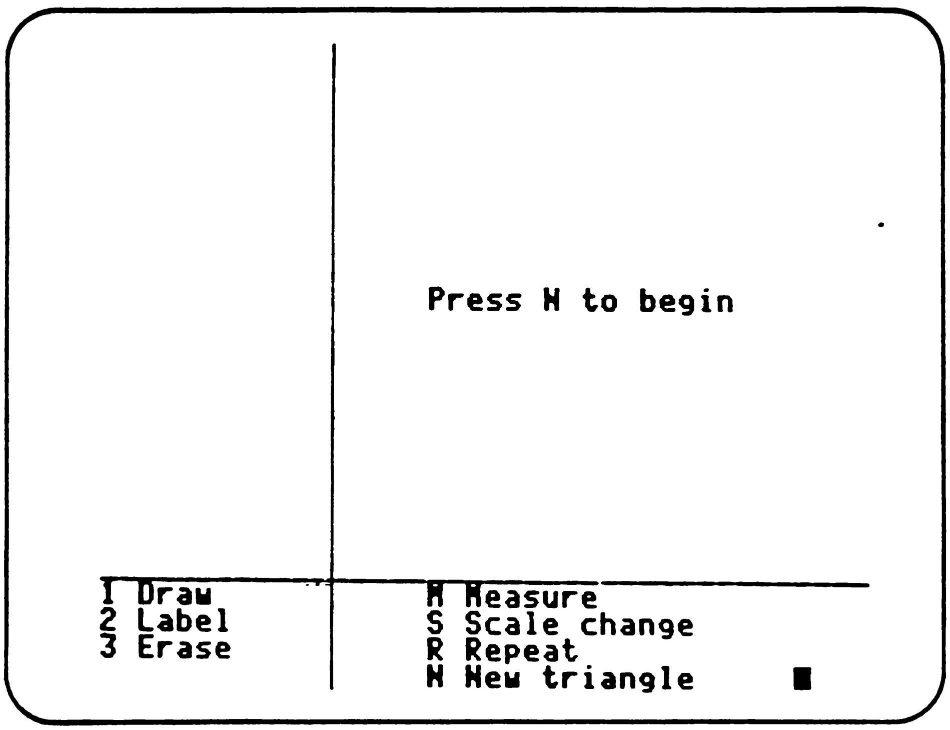
After more than three years of living with an intellectual undertaking that one nurtures and develops, it is difficult to say it is finished; but it is. However, the beliefs that led us to undertake development of the SUPPOSER can be articulated in other domains of mathematics. We turn eagerly now to that task.

Judah L. Schwartz
Michal Yerushalmy

I.1 HOW DO I GET STARTED?

Put THE GEOMETRIC SUPPOSER disk into the disk drive, close the door of the drive and turn on the computer and the monitor. The program will take about 30 seconds to load. During that time, the title screen will be displayed.

After the program has been loaded into memory, the screen will look like this:



The screen of the SUPPOSER is divided into three sections: the data column on the left side, the construction pad on the right side, and a window for menus and prompts at the bottom, below the horizontal line.

The message at the bottom of the screen is:

1 Draw	M Measure
2 Label	S Scale change
3 Erase	R Repeat
	N New triangle

This is the MAIN MENU of the program and the one to which you will be returning over and over again. As indicated on the screen, to begin you must press N for New Triangle in order to proceed. At this point the SUPPOSER will not respond to any other key.

I.2 HOW DO I MAKE A TRIANGLE?

After you press N, the message at the bottom of the screen will change to:

1 Right	4 Isosceles
2 Acute	5 Equilateral
3 Obtuse	6 Your own

Choices 1 through 5 allow you to generate a random right, acute, obtuse, isosceles, or equilateral triangle, respectively. Choice 6 allows you to define a triangle of your own making on which to make your constructions.

If you choose option 2, for example, an acute triangle labeled ABC will be drawn, and the MAIN MENU will be displayed at the bottom of the screen. You can now carry out constructions on the triangle.

If you choose option 6, Your own, the message at the bottom of the screen will change to:

1 Side	2 Side	3 Angle
Side	Angle	Side
Side	Side	Angle

0 < Side < 9

There are three different ways you may specify your triangle. They are:

1. specifying the length of the three sides of the triangle

2. specifying the length of two sides of the triangle and the size of the angle included by those sides

3. specifying the size of two angles and the length of the side included by those angles

Select one of these options by pressing the corresponding number and the prompts for specifying your triangle will appear at the bottom of the screen.

Also a unit length will be drawn in the upper right-hand corner of the screen, labeled with the letter u. This length is the standard unit of length to be used in multiples for all constructions. (We suggest using sides with lengths less than about 8 units in order to be sure that your triangle will fit on the screen. The screen is about 10 units high and the location

of the first vertex to be drawn is placed roughly in the center of the screen. If your triangle turns out to be too small, you can then use the rescale option--see Scale Change, pp. 75.)

In defining.the lengths of sides and the sizes of angles, you may enter values to two decimal places.

Suppose you wish to construct a triangle by specifying two angles of 45 degrees and 75 degrees and a side of length 8 to be between them. After entering the number 6, Your own, press 3 for angle-side-angle, and the following message will appear

angle BAC =
side AB
angle CBA

asking you to supply the values of angle BAC, side AB, and angle CBA. Enter the first angle, BAC (45 degrees, in this case), and press RETURN. The SUPPOSER will display an angle of 45 degrees formed by two rays that go out to the edge of the screen. Enter the side length, AB (8, in this case), and press RETURN. The SUPPOSER will lay off an arc across the rays. Finally, enter the second angle, CBA (75 degrees in this case), and press RETURN. The SUPPOSER will construct that angle from the end of the line segment of length 8. You will then be presented with two options at the bottom of the screen:

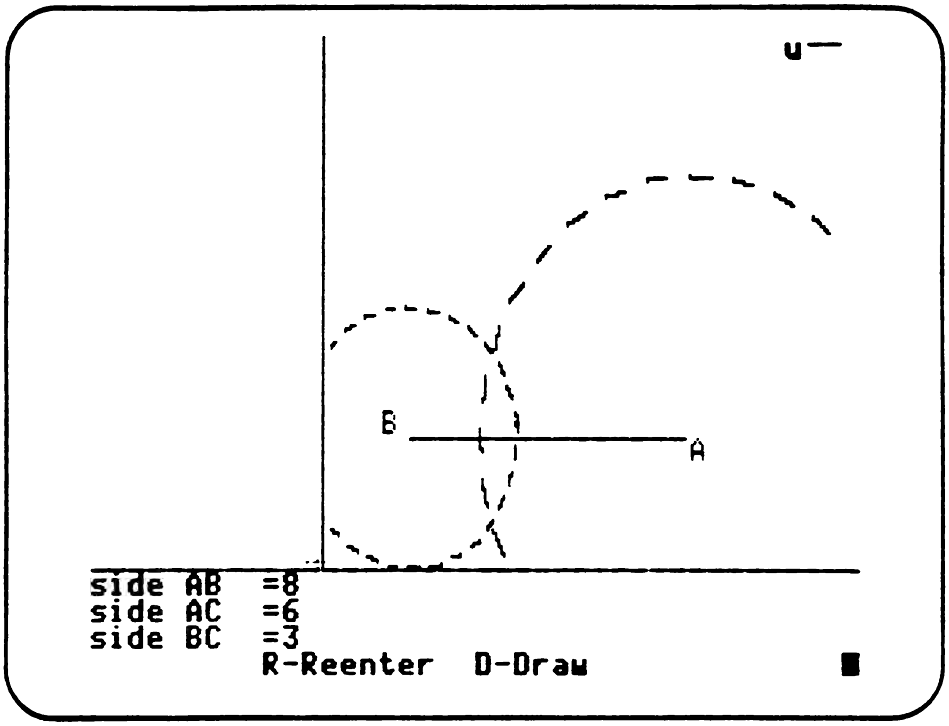
R - Reenter   D - Draw

R (Reenter) allows you to start over and to specify a new triangle by returning you to the menu of options for constructing your own triangle.

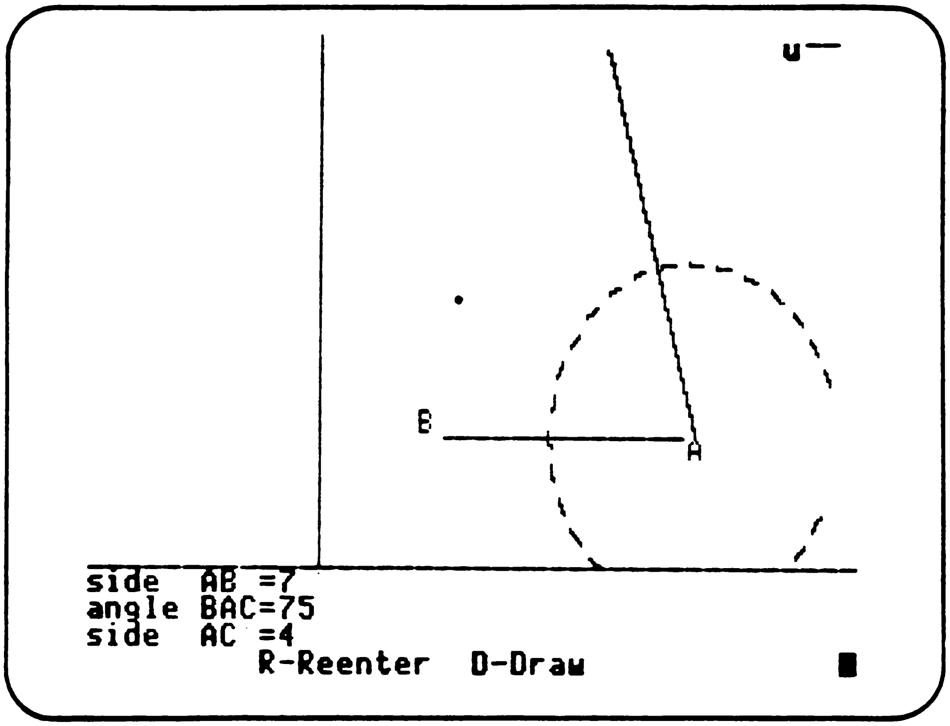
D (Draw) will cause the SUPPOSER to draw the triangle you have just defined and will label it. Press RETURN and the construction rays and arcs will be erased and you will be returned to the MAIN MENU. You are now ready to carry out constructions on your triangle.

Should your triangle be too large to fit on the screen, the SUPPOSER will return to the beginning of the Your Own triangle menu.

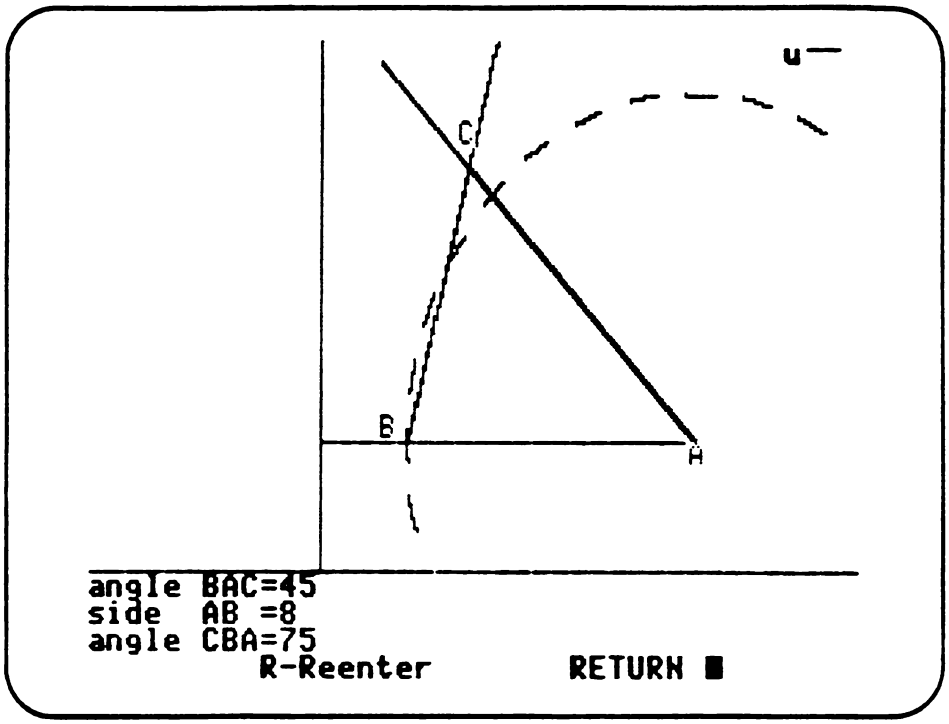
Here is the construction of a triangle by SIDE SIDE SIDE:



Here is the construction of a triangle by SIDE ANGLE SIDE:



Here is the construction of a triangle by ANGLE SIDE ANGLE:



I.3 THE GEOMETRIC SUPPOSER--AN OVERVIEW OF THE MAIN MENU

The MAIN MENU of the program displays across the bottom of the screen:

1 Draw	M Measure
2 Label	S Scale change
3 Erase	R Repeat
	N New triangle

To choose one of these options, enter the corresponding number or letter and you will be presented with the appropriate submenu.

In this section, each of these options will be described briefly to give you an overview of THE GEOMETRIC SUPPOSER. In the next section of the manual, each of the options and the suboptions will be described in much greater detail.

Pressing ESC returns you to the previous menu.

1 Draw

If you choose option 1 in the MAIN MENU, Draw, the message at the bottom of the screen changes to:

1 Segment	5 Parallel	9 Midsegment
2 Circle	6 Perpendicular	0 Extension
3 Median	7 Angle bisect	
4 Altitude	8 Perpendicular bisect	

These are the possible constructions that you may draw.

To draw one of these constructions, enter the appropriate number and the submenu for that construction will appear on the screen.

The operation of each of these choices is explained in detail in the next section of the manual.

Pressing ESC returns you to the previous menu.

2 Label

If you choose option 2 in the MAIN MENU, Label, the message at the bottom of the screen changes to:

1 Intersection
2 Subdivide segment
3 Reflection
4 Random Point

This option allows you to label intersections, subdivide line segments, reflect portions of a figure in a line, and generate a random point.

To carry out one of these operations, enter the appropriate number and the submenu will appear on the screen.

The operation of each of these choices will be explained in detail in the next section of the manual.

Pressing ESC returns you to the previous menu.

3 Erase

If you choose option 3 in the MAIN MENU, Erase, the message at the bottom of the screen changes to:

1 Erase segment:
2 Erase label(s):

This option allows you to clear a segment or a label from the screen. It also permits you to unclutter a cluttered construction by erasing labels that you may not need at the moment. Once erased, segments and labels will not appear on the screen; they remain in the memory, however, and can be used to carry out constructions and measurements.

The operation of each of these choices is explained in detail in the next section of the manual.

Pressing ESC returns you to the previous menu.

M Measure

If you choose option M on the MAIN MENU, Measure, the message at the bottom of the screen changes to:

1 Length	5 Distance Point-Line
2 Perimeter	6 Distance Line-Line
3 Area	7 Adjustable element(s)
4 Angle	

This option allows you to measure lengths, perimeters, areas, and angles; and the distances between points and lines, and between lines and lines. You can also measure the sum, difference, product, or ratio of two lengths, perimeters, areas, angles, or distances. You can square the measure of a length, perimeter, area, angle, or distance as well. The values of the measured quantities are printed in the Data Column on the left-hand side of the screen.

> To take one of these measurements, enter the appropriate number and the submenu will appear on the screen.

The operation of each of these choices is explained in detail in the next section of the manual.

Pressing ESC returns you to the previous menu.

S Scale Change

There are two sizes available for each image. If you choose option S in the MAIN MENU, Scale Change, the image on the screen is redrawn in the other of the two possible sizes.

This option has another function as well. If a construction cannot be fully displayed on the screen and the beep sounds, try using the Scale Change option. This option, in addition to changing the scale of the triangle, checks the placement of the triangle on the screen. In most cases, the program can relocate the triangle so that the construction can be displayed.

To carry out a scale change, enter the letter S and the SUPPOSER will rescale the triangle.

R Repeat

If you choose this option in the MAIN MENU, the message across the bottom of the screen will change to:

Repeat construction:
1 on new triangle
2 on previous triangle

You may now repeat the construction you have just made on a new triangle or on any one of the three previous triangles.

The operation of each of these options is explained in detail in the next section of the manual.

Pressing ESC returns you to the previous menu.

N New triangle

If you choose this option in the MAIN MENU (by pressing N), then you will be returned to the triangle construction menu described in Section I.2 (HOW DO I MAKE A TRIANGLE?).

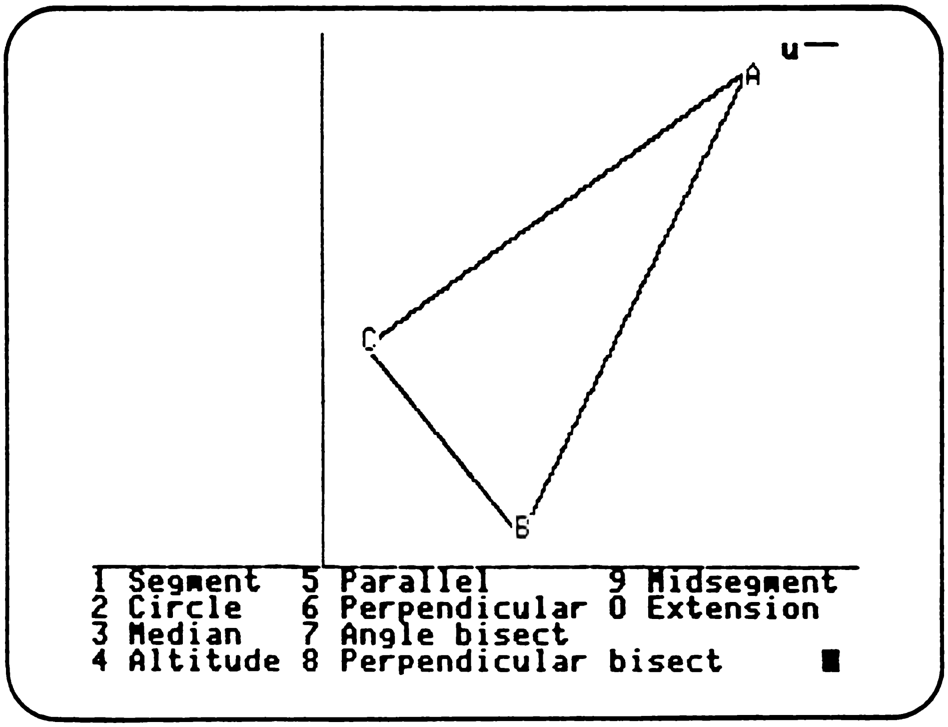
II. USING THE GEOMETRIC SUPPOSER--THE SUBMENUS

The heart of using THE GEOMETRIC SUPPOSER lies in its submenus. This section of the manual describes each choice in each submenu of the MAIN MENU.

At this point we suggest that you refer to the reference card included with this program; it summarizes the various submenus, and gives you essential information about the operation of the SUPPOSER.

DRAW

After you have created a triangle, you can make constructions on that triangle by using the Draw option. Selecting Draw will result in the following submenu screen:



1 Segment

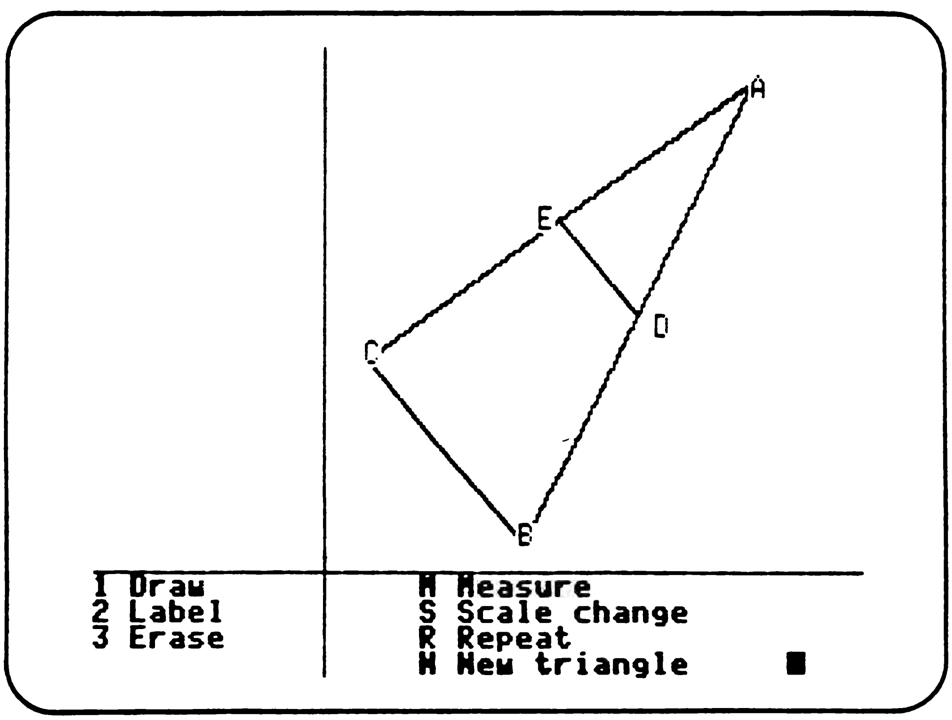
Choice 1 under Draw is Segment. If you choose this option, the SUPPOSER will ask what segment you want to draw. You may connect any two labeled points on the screen.

After entering the number 1, the message at the bottom of the screen will read

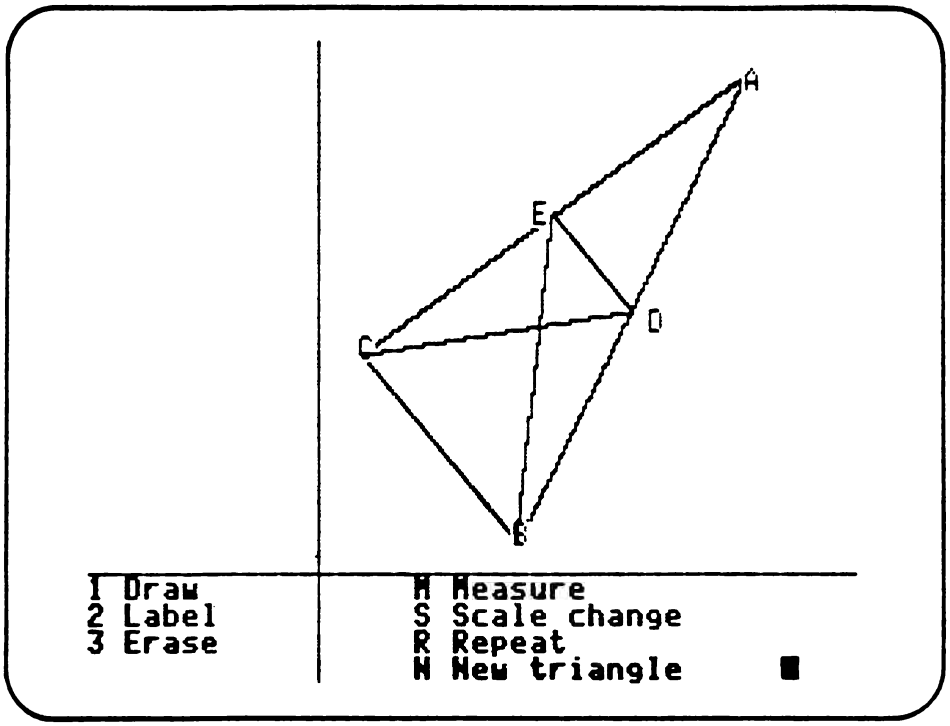
Segment name:

asking you to identify the names of the two points you want to connect with a segment. Enter the name of the segment and the SUPPOSER will draw the segment and return you to the MAIN MENU.

For example, suppose you have made the construction shown below in triangle ABC:



You may use this option to draw segments BE and DC as shown below:



2 Circle

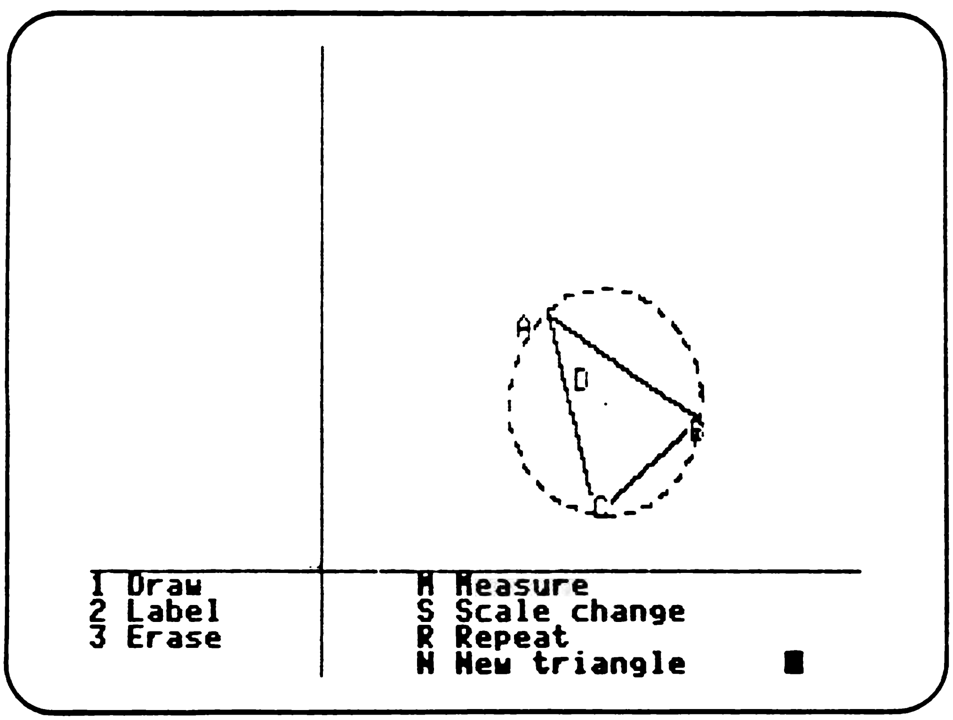
Choice 2 under Draw is Circle. If you choose this option, the SUPPOSER will ask you whether you want to draw a circumscribed or inscribed circle, or to define your own circle by specifying the center of the circle and the length of the radius.

After entering the number 2, the message on the screen will read:

1 Circumscribes triangle:
2 Inscribed in triangle:
3 Other

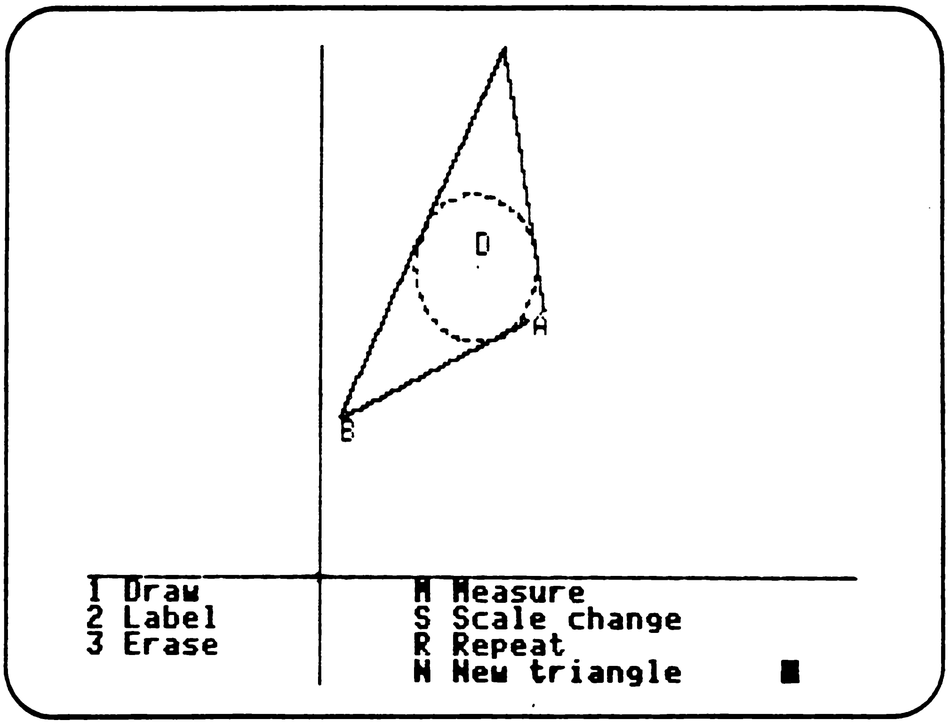
If you choose Circumscribes triangle, enter the name of the triangle which you would like the circle to circumscribe (e.g., ABC). The SUPPOSER will draw the circle, label its center (D), and return you to the MAIN MENU.

Here is a circle that circumscribes an isosceles right triangle:



If you choose Inscribed in triangle, enter the name of the triangle in which you would like to draw an inscribed circle (e.g., ABC). The SUPPOSER will draw the circle, label its center (D), and return you to the MAIN MENU.

Here is a circle inscribed in an isosceles triangle:



Choice 3, Other, allows you to draw a circle with any labeled point on the screen as its center and with a radius of your selection.

For example, suppose you wish to construct a circle on triangle ABC, with point B serving as its center, and with a radius equal to side AB.

After entering the number 3, the following message will appear on the screen:

Circle's center:

Now enter the name of the point that will serve as the center of the circle (B).

Any labeled point on the screen may serve as the center of the circle. If the point that you wish to be the center of your circle has no label, you may place a point there and label it. See the options in the Label menu.

After you identify the center of your circle (in this case, B), you must specify the length of the radius of your circle.

You can specify the radius in terms of 1) some constant multiple of the length of any segment depicted on the screen, or 2) some multiple of the unit length u which appears in the upper right-hand corner of the screen.

The message on the screen will now read:

```
Circle's center: B
Radius =(segment or unit) * constant
       =                  *
```

To construct a circle with a radius equal to the length of AB, enter AB for segment or unit. The message will now read:

```
Circle's center: B
Radius =(segment or unit) * constant
       = AB               *
```

Now for constant, enter 1, press RETURN, and the SUPPOSER will draw the circle you have defined and return you to the MAIN MENU.

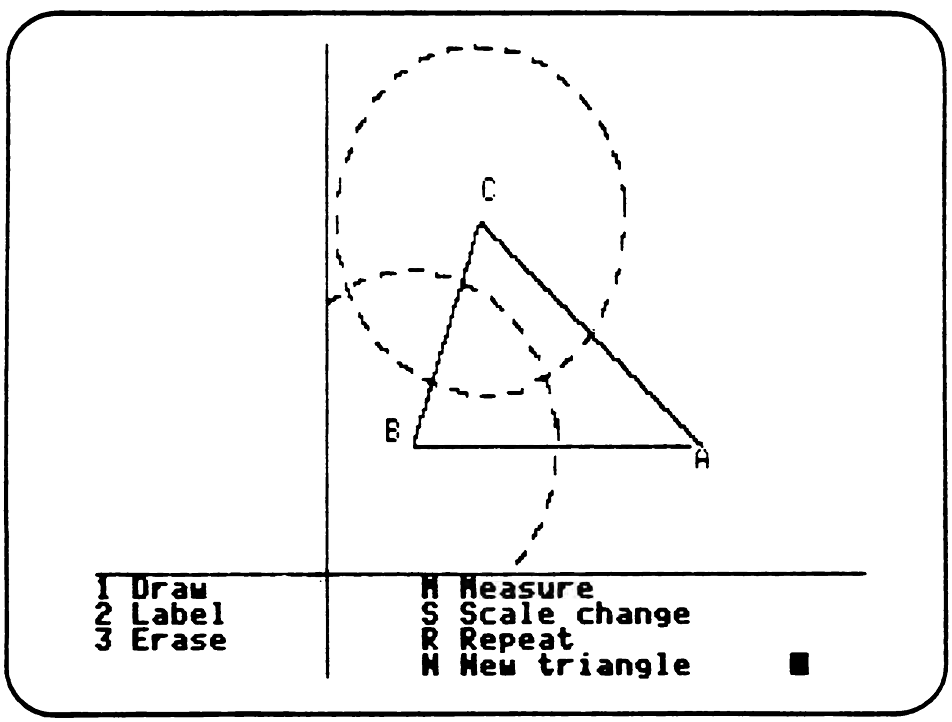
If instead you want to define the radius in terms of some multiple of the unit length u, enter u for segment or unit and then for constant, enter the multiplier that you want to use. Press RETURN and the SUPPOSER will draw the circle.

To illustrate the distinction between these two ways of specifying a radius, consider the following example and figures. Triangle ABC is an isosceles triangle with side lengths AB = AC = 8. Suppose we draw a circle centered on B with a radius of AB * .5, and a circle of radius 4 centered on C.

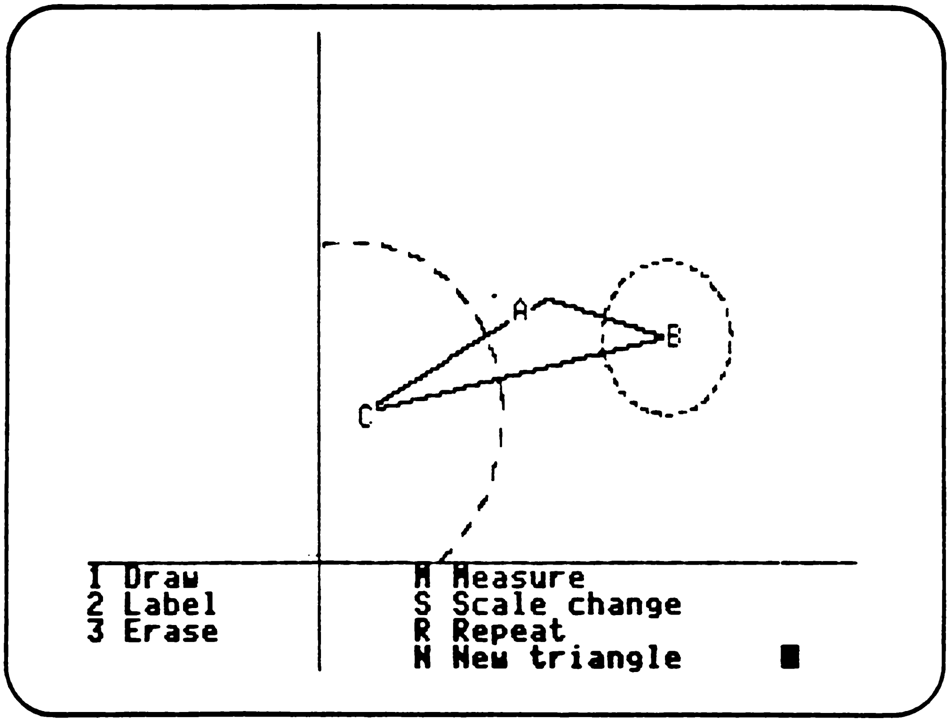
The radius of the circle centered on B is defined in terms of the length of the segment AB. The radius of the circle centered on C is defined in terms of a certain number of screen units. Even though the two radii have the same numerical value, when we repeat the construction on a different triangle, or if we rescale the drawing, the two circles will behave differently.

If the construction of the triangle and the two circles is repeated, the radius of the circle centered on B remains one-half of the length of the segment AB, while the radius of the circle centered on C is still 4 units.

Here are the circles constructed on isosceles triangle ABC:



In this figure, the construction of the circles has been repeated on an obtuse triangle. Note the difference in the size of the circle centered on B (with its radius defined in terms of AB) when the construction is repeated.



3 Median

Choice 3 under Draw is Median. The SUPPOSER will ask you to name the triangle in which the median is to be drawn and from which vertex it is to be drawn.

Suppose you want to construct a median in triangle ABC from vertex B. After entering 3, Median, the message on the screen will read:

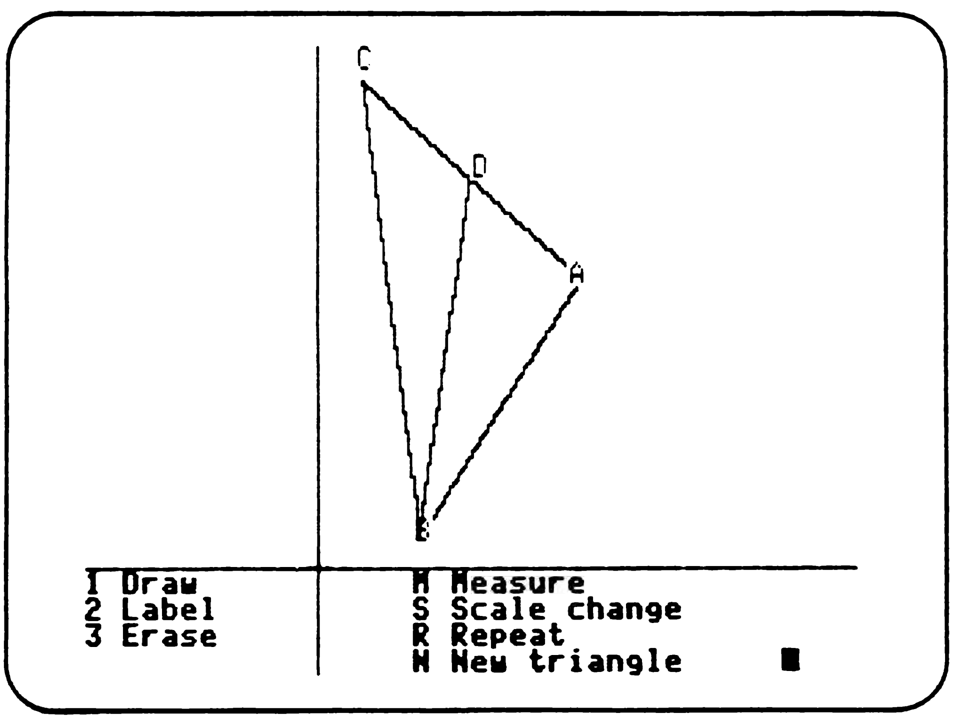
Median in triangle:

Enter the name of the triangle (ABC), and the message will now read:

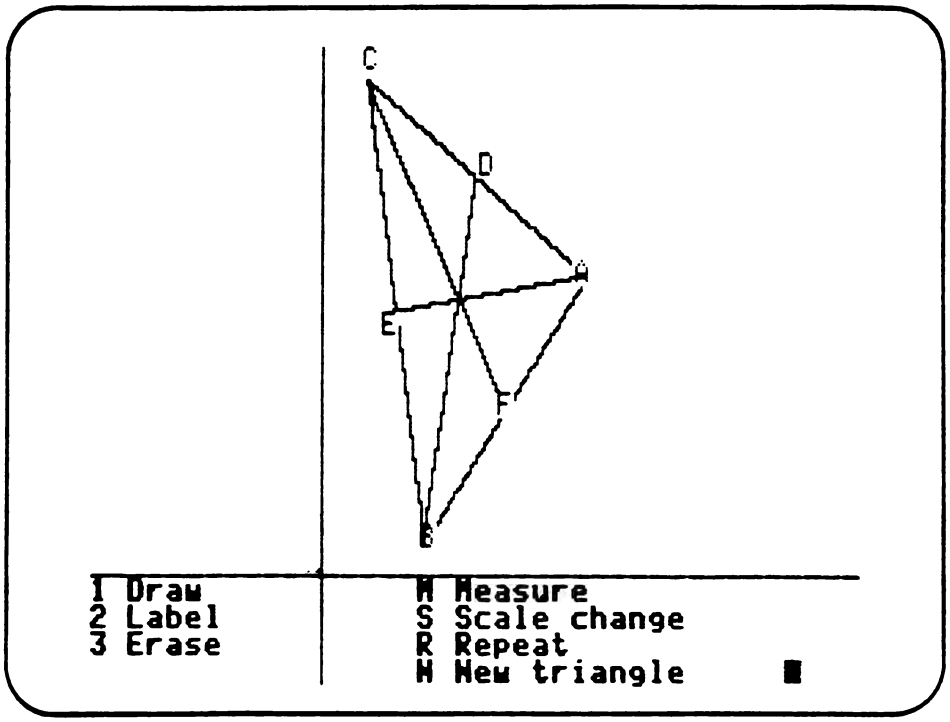
Median in triangle: ABC
From vertex:

Enter the name of the vertex (B). The SUPPOSER will draw the median from vertex B in triangle ABC, label the endpoint of the median as D (or the next available letter), and return you to the MAIN MENU.

Here is triangle ABC with a median drawn from B:



Here are the three medians of triangle ABC:



4 Altitude

Choice 4 under Draw is Altitude. The SUPPOSER will ask you which triangle the altitude lies in and from which vertex it is to be drawn.

Suppose you want to draw an altitude in triangle ABC from vertex B. After entering the number 4, Altitude, the message on the screen will read:

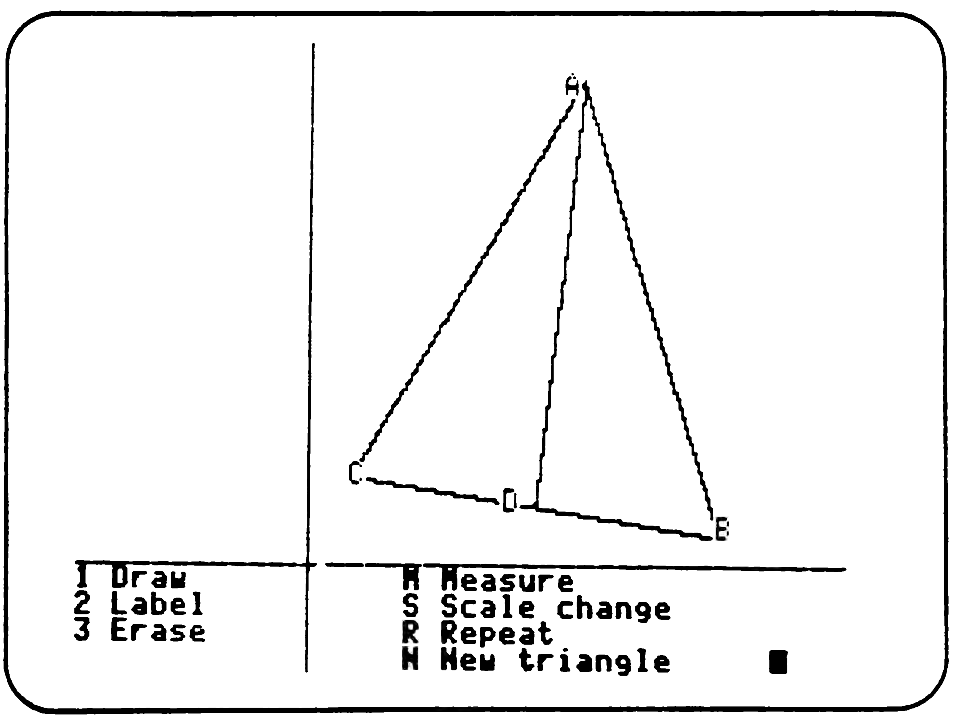
Altitude in triangle:

Enter the name of the triangle (ABC) and the message will now read:

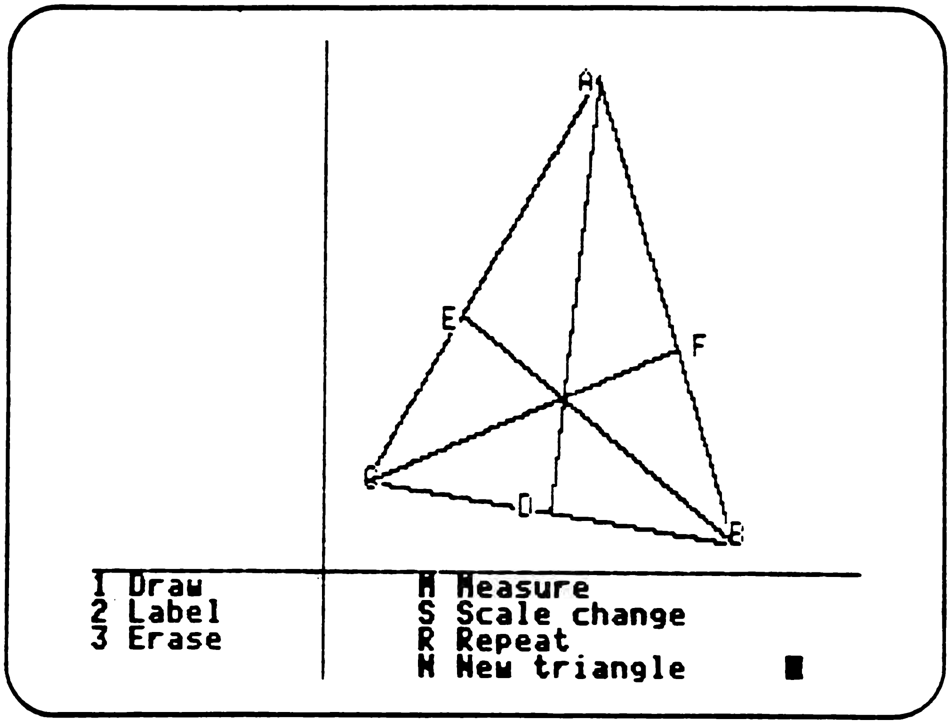
Altitude in triangle: ABC
From vertex:

Enter the name of the vertex (B) and the SUPPOSER will draw the altitude from vertex B in triangle ABC, labeling the endpoint of the altitude as D, and will return you to the MAIN MENU.

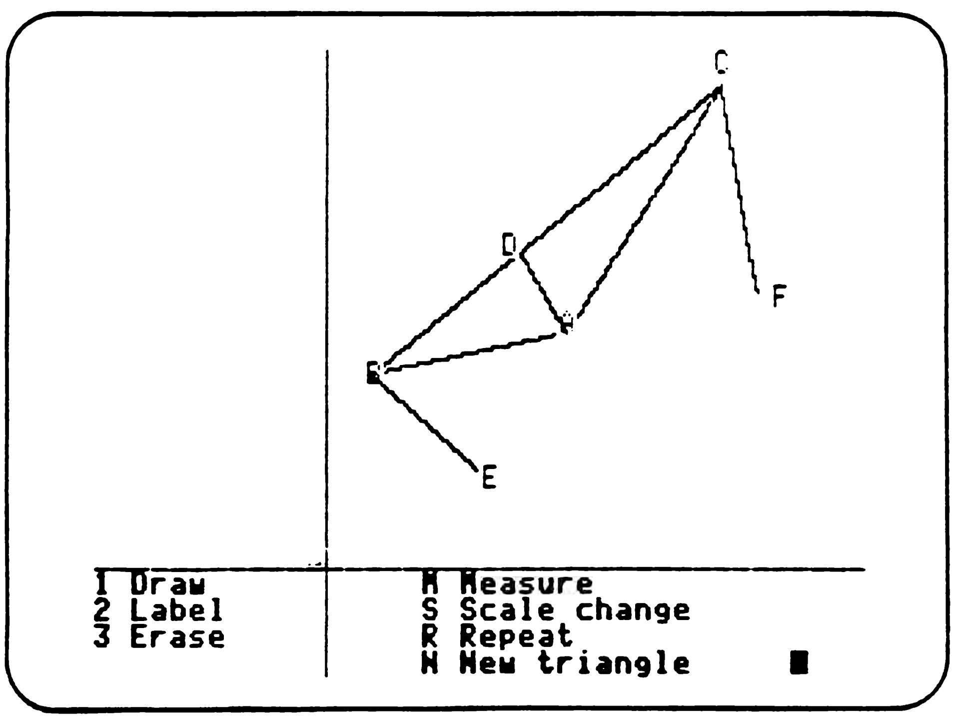
Here is an altitude in triangle ABC:



Here are the three altitudes of an acute triangle:



Here are the three altitudes of an obtuse triangle:



5 Parallel

Choice 5 under Draw is Parallel. Choosing this option allows you to draw a line parallel to any line segment on the screen. The SUPPOSER will first ask through what point the parallel is to be drawn. It will then ask which segment it will parallel. You will then be asked to specify the length of the parallel 1) in terms of some segment or the unit length, or 2) to intersect a segment on the screen.

Suppose you want to draw a parallel through point B in triangle ABC, parallel to side AC, with a length equal to side AB. After entering the number 5, Parallel, the message on the screen will read:

```
Parallel through point:
```

Enter the name of the point through which you want the parallel to pass (B). The message will now read:

```
Parallel through point:  B
Parallel to line:
```

Enter the name of the segment that you want to parallel (AC). The message on the screen will now read:

```
Length defined:
1 by (segment or unit) * constant
2 to intersect segment(s)
```

Select option 1 to define the length of the parallel by a segment or unit length. The message will now read:

```
Length defined:
      (segment or unit) * constant
                        *
```

To construct a parallel with a length equal to segment AB, enter AB under segment or unit. The message will now read:

```
Length defined :
     (segment or unit)  * constant =
         AB             *
```

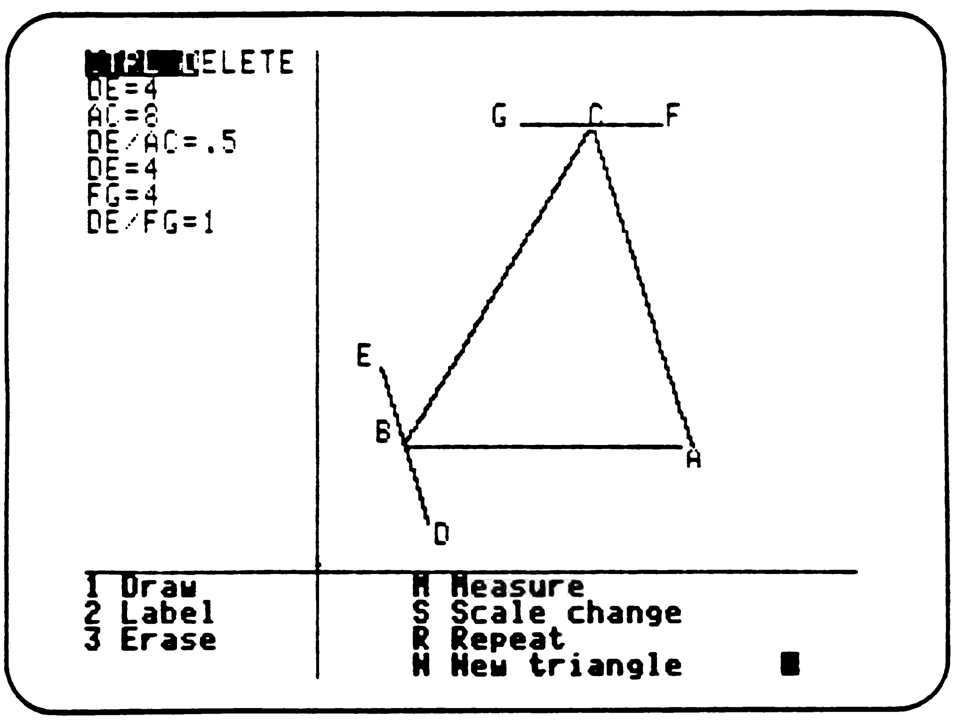
Now enter 1 for the constant and press RETURN. The SUPPOSER will draw the parallel, label the endpoints DE, and return you to the MAIN MENU.

If instead you want to define the parallel in terms of some multiple of the unit length u , enter u for segment or unit and then for constant, enter the multiplier that you want to use. Press RETURN and the SUPPOSER will draw and label the parallel.

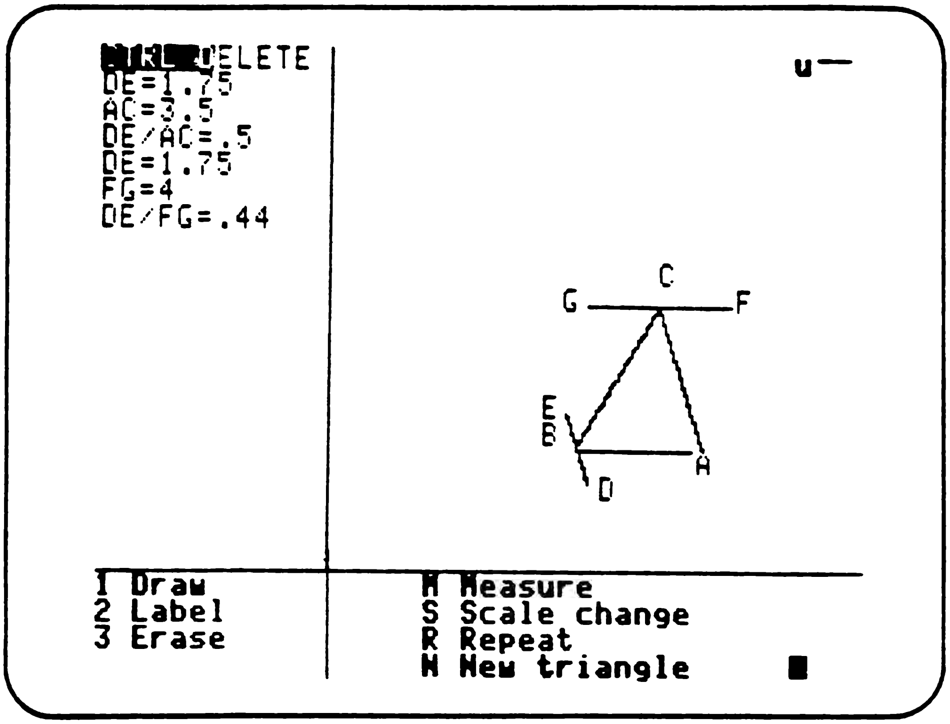
To illustrate the distinction between these two ways of specifying the length of a parallel, consider the following example and figures. Triangle ABC is an isosceles triangle with side lengths AB = AC = 8. Suppose we draw a parallel through B, parallel to AC. Let the length of this parallel be AC * .5. Suppose, in addition, we draw a parallel through C, parallel to AB. Let the length of this parallel be 4 units long.

The length of the parallel drawn through B is defined in terms of the length of the segment AC. The length of the parallel drawn through C is defined in terms of a certain number of screen units. Even though the lengths of the two parallels have the same numerical value, when we repeat the construction on a different triangle, or if we rescale the drawing, the two parallels will behave differently.

Here are the parallels drawn on triangle ABC:



In this figure, the constructions have been repeated on another triangle. Note the difference in length of the parallel drawn through point B (defined in terms of segment AC) when the construction is repeated.



The second way to specify the length of the parallel, <u>to intersect segment(s)</u>, allows you to extend a parallel until it intersects the line segment or segments you name. When the message on the screen reads

Length defined:
1 by (segment or unit) * constant
2 to intersect segment(s)

enter 2, then the name of the segment you want the parallel to intersect. Press RETURN and the SUPPOSER will draw the parallel to intersect that segment. If you wish to draw the parallel to intersect two segments, enter the name of the first segment, then enter the name of the second segment, and the SUPPOSER will draw the parallel to intersect the two segments that you have named and label the intersections.

6 Perpendicular

Choice 6 under Draw is Perpendicular. Choosing this option allows you to draw a line perpendicular to any line segment. The SUPPOSER will first ask through what point the perpendicular is to be drawn and then to which segment it will be perpendicular. You will then be asked to specify the length of the perpendicular 1) in terms of some segment or the unit length, or 2) to intersect segment(s) on the screen.

Suppose you want to draw a perpendicular through point B in triangle ABC, perpendicular to side AC, with a length equal to side AB. After entering 6, Perpendicular, the message on the screen will read:

```
Perpendicular through point:
```

Enter the name of the point through which you want the perpendicular to pass (B). The message will now read:

```
Perpendicular through point:  B
Perpendicular to line:
```

Enter the name of the segment to which it will be perpendicular (AC). The message on the screen will now read:

```
Length defined:
1 by (segment or unit)  * constant
2 to intersect segment(s)
```

Select option 1 to define the length of the perpendicular by a segment or the unit length. To construct a perpendicular with a length equal to segment AB, enter AB for segment or unit. The message will now read:

```
Length defined:
     (segment or unit)  * constant =
         AB             *
```

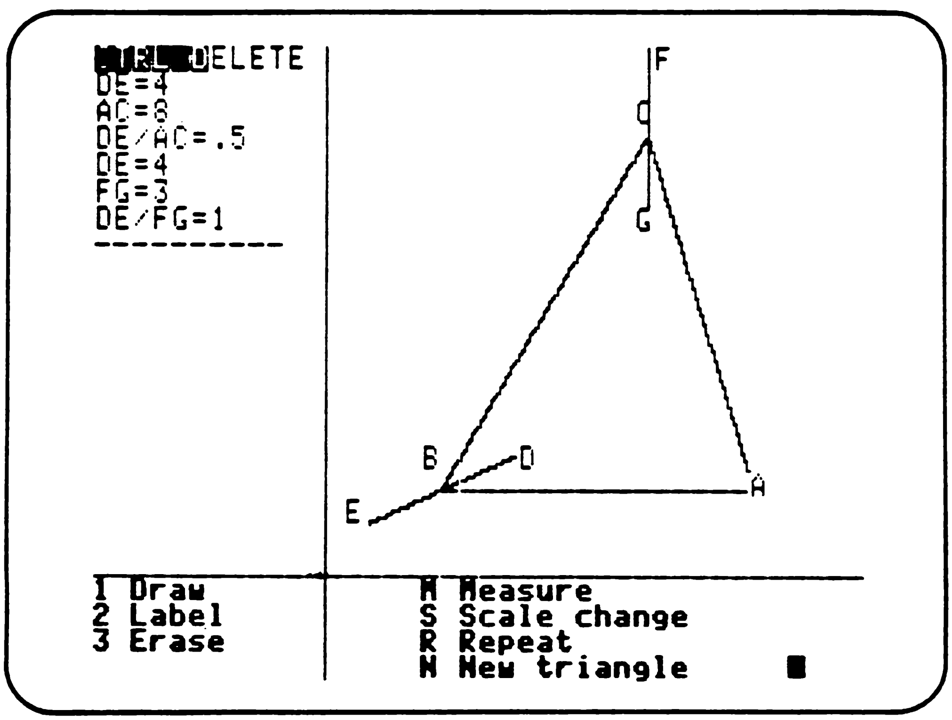
Now enter 1 for the constant and press RETURN. The SUPPOSER will draw the perpendicular, label the endpoint, and return you to the MAIN MENU.

If instead you want to define the perpendicular in terms of some multiple of the unit length u, enter u for segment or unit and then for constant, enter the multiplier that you want to use. Press RETURN and the SUPPOSER will draw the perpendicular, label it, and return you to the MAIN MENU.

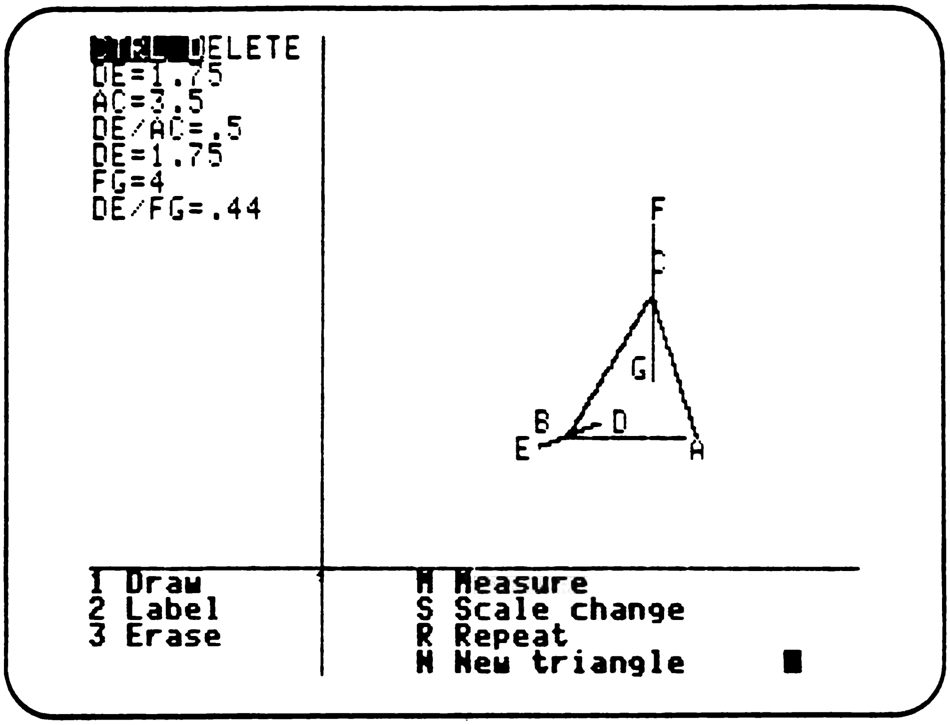
To illustrate the distinction between these two ways of specifying the length of a perpendicular, consider the following example and figures. Triangle ABC is an isosceles triangle with side lengths AB = AC = 8. Suppose we draw a perpendicular through B, perpendicular to AC. Let the length of this parallel be AC * .5. Suppose, in addition, we draw a perpendicular through C, perpendicular to AB. Let the length of this perpendicular be 4 units long.

The length of the perpendicular drawn through B is defined in terms of the length of the segment AC. The length of the perpendicular drawn through C is defined in terms of a certain number of screen units. Even though the lengths of the two perpendiculars have the same numerical value, when we repeat the construction on a different triangle, or if we rescale the drawing, the two perpendiculars will behave differently.

Here are the perpendiculars constructed on isosceles triangle ABC:



In this figure, the same perpendiculars have been repeated on another triangle. Note the difference in length of the perpendicular drawn through B (defined in terms of AC) when the construction is repeated.



The second way to specify the length of the perpendicular, <u>to intersect segment(s)</u>, allows you to extend a perpendicular until it intersects the line segment or segments you name. When the message on the screen reads

> Length defined:
> 1 by (segment or unit) * constant
> 2 to intersect segment(s)

enter 2, then the name of the segment you want the perpendicular to intersect. Press RETURN and the SUPPOSER will draw the perpendicular to intersect that segment. If you wish to draw the perpendicular to intersect two segments, enter the name of the first segment, then enter the name of the second segment. The SUPPOSER will draw the perpendicular to intersect the two segments that you have named, label them, and return you to the MAIN MENU.

7 Angle bisector

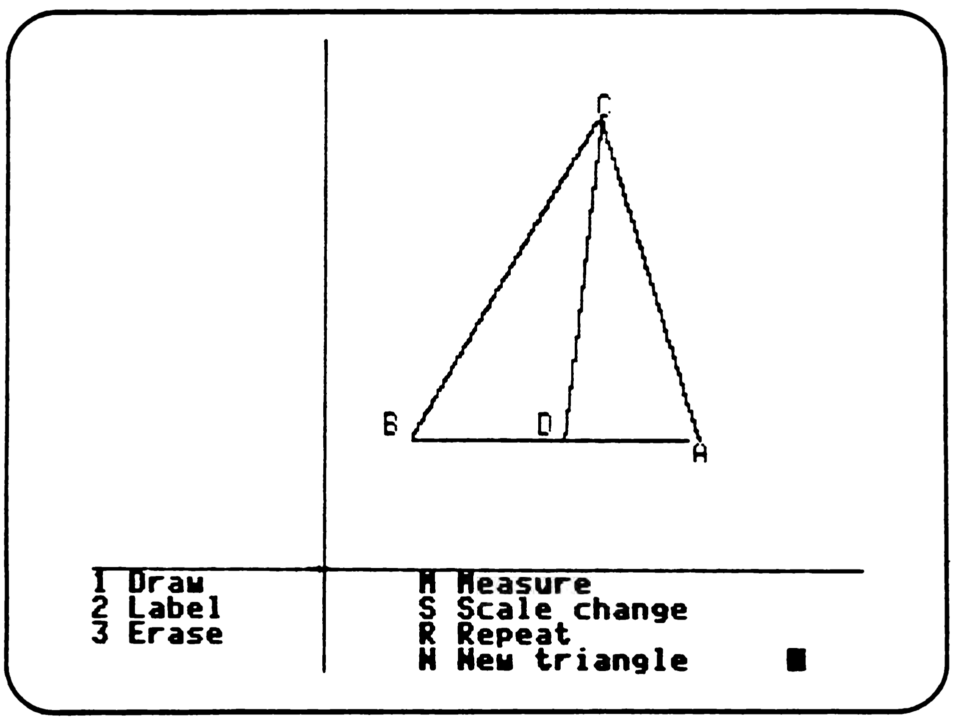
Choice 7 under Draw is Angle bisector. Choosing this option allows you to draw the bisector of any angle on the screen. The SUPPOSER will ask for the name of the angle to be bisected.

Suppose you want to bisect angle BCA in triangle ABC. After entering the number 7, the message on the screen will read:

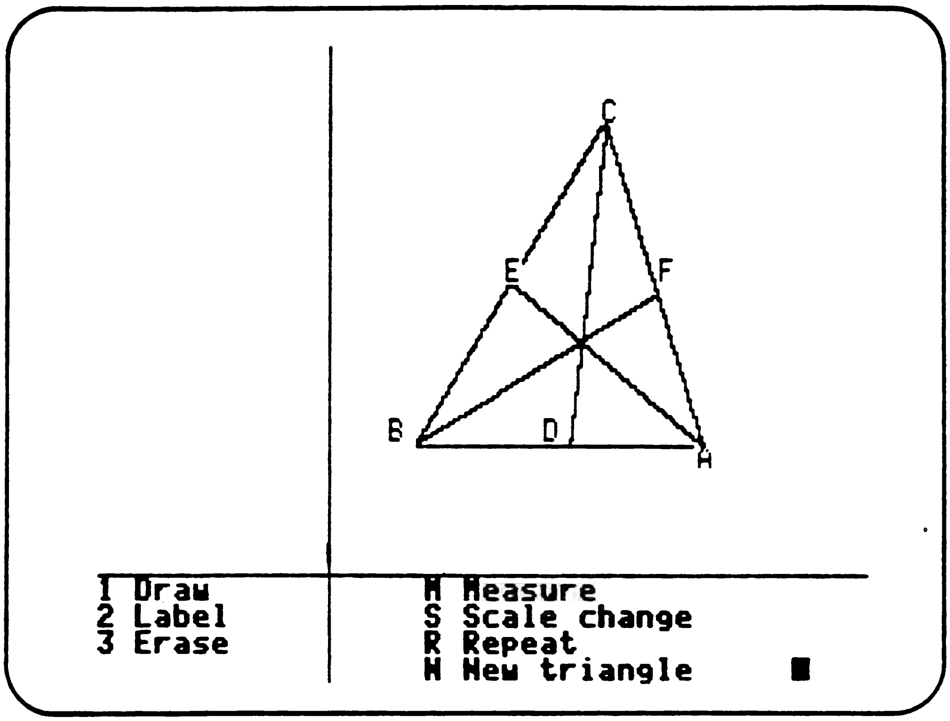
Bisect angle:

Enter the name of the angle (BCA) to be bisected and the SUPPOSER will draw the bisector of angle BCA and label the point where it intersects the opposite side as D.

Here is triangle ABC with the bisector of angle BCA drawn:



Here is the same triangle with all its angle bisectors drawn:



8 Perpendicular bisector

Choice 8 under Draw is Perpendicular bisector. Choosing this option allows you to draw a perpendicular bisector to any line segment on the screen. The SUPPOSER will ask you to specify which segment you wish the perpendicular to bisect and then to define the length of the bisector 1) in terms of a segment or the unit length, or 2) to intersect segment(s) on the screen.

Suppose you want to draw the perpendicular bisector of side AC in triangle ABC, with a length equal to side AB. After entering the number 8, the message on the screen will read:

Perpendicular bisect to segment:

Enter the name of the segment (AC) and the message will now read:

Length defined:
1 by (segment or unit) * constant
2 to intersect segment(s)

Select option 1 to define the length of the bisector by a segment or the unit length. To draw the bisector of side AC with a length equal to segment AB, enter AB for segment or unit. The message will now read:

Length defined:
(segment or unit) * constant =
AB *

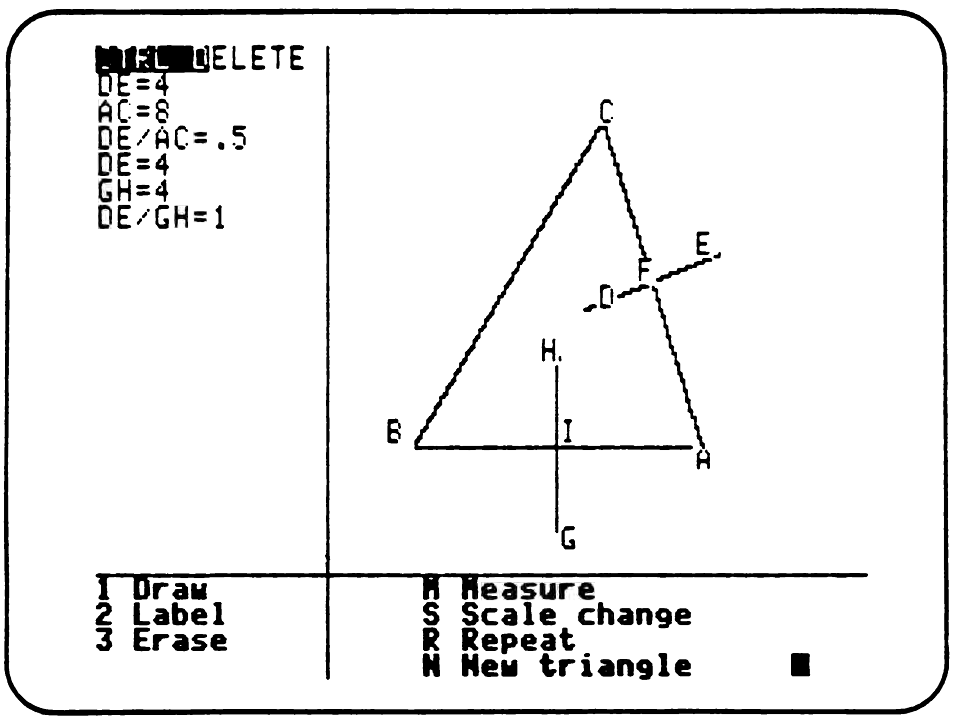
Now enter 1 for the constant and press RETURN. The SUPPOSER will draw the bisector and label it.

If instead you want to define the bisector in terms of some multiple of the unit length u , enter u for segment or unit and then for constant, enter the multiplier that you want to use. Press RETURN and the SUPPOSER will draw the bisector and label it.

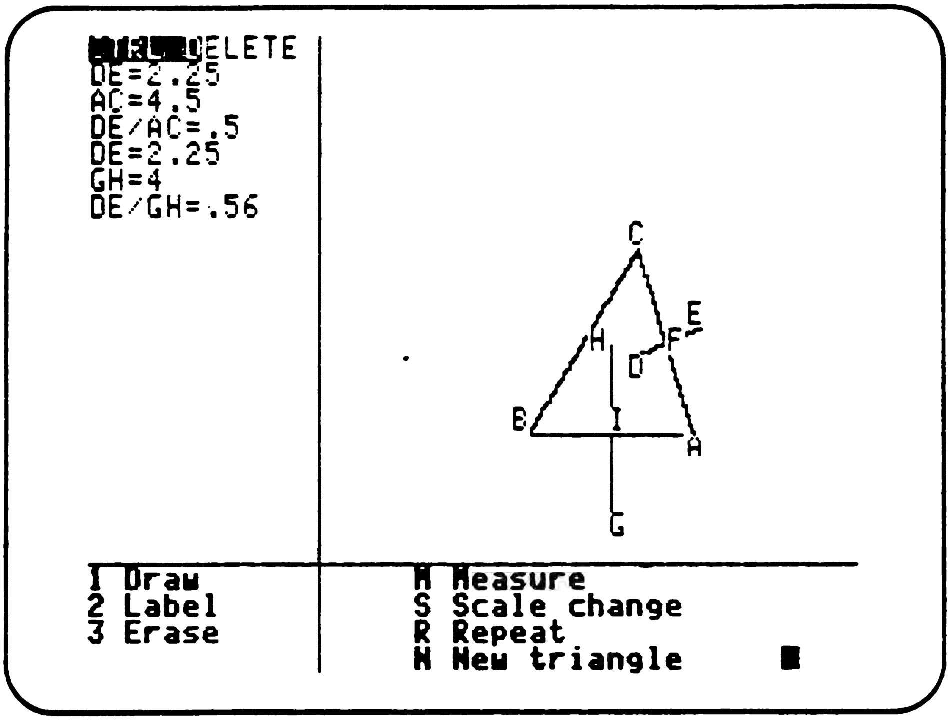
To illustrate the distinction between these two ways of specifying the length of a perpendicular bisector, consider the following example and figures. Triangle ABC is an isosceles triangle with side lengths AB = AC = 8. Suppose we draw a perpendicular bisector of AC. Let the length of this perpendicular bisector be AC * .5. Suppose, in addition, we draw a perpendicular bisector of AB. Let the length of this perpendicular bisector be 4 units long.

The length of the perpendicular bisector of AC is defined in terms of the length of the segment AC. The length of the perpendicular bisector of AB is defined in terms of a certain number of screen units. Even though the lengths of the two perpendicular bisectors have the same numerical value, when we repeat the construction on a different triangle, or if we rescale the drawing, the two perpendicular bisectors will behave differently.

Here are the perpendicular bisectors drawn in the isosceles triangle:



In this figure, the perpendicular bisectors are repeated on another triangle. Note the difference in the length of the perpendicular bisector of AC (defined in terms of the length AC) when the construction is repeated.



The second way to specify the length of the perpendicular bisector, to intersect segment(s), allows you to draw a perpendicular bisector until it intersects the named line segment or segments. When the message on the screen reads

> Length defined:
> 1 by (segment or unit) * constant
> 2 to intersect segment(s)

enter 2, then the name of the segment you want the perpendicular bisector to intersect. Press RETURN and the SUPPOSER will draw the perpendicular bisector to intersect that segment. If you wish to draw the perpendicular bisector to intersect two segments, enter the name of the first segment, then enter the name of the second segment. The SUPPOSER will draw the perpendicular bisector to intersect the two segments that you have named.

9 Midsegment

Choice 9 under Draw is Midsegment. Choosing this option allows you to draw a line segment connecting the midpoints of two line segments. The SUPPOSER will ask you to name the segment from which the midsegment is to be drawn and the segment to which the midsegment is to be drawn.

Suppose you want to draw the midsegment from the midpoint of segment AB to the midpoint of segment BC in triangle ABC. After entering the number 9, the message on the screen will read:

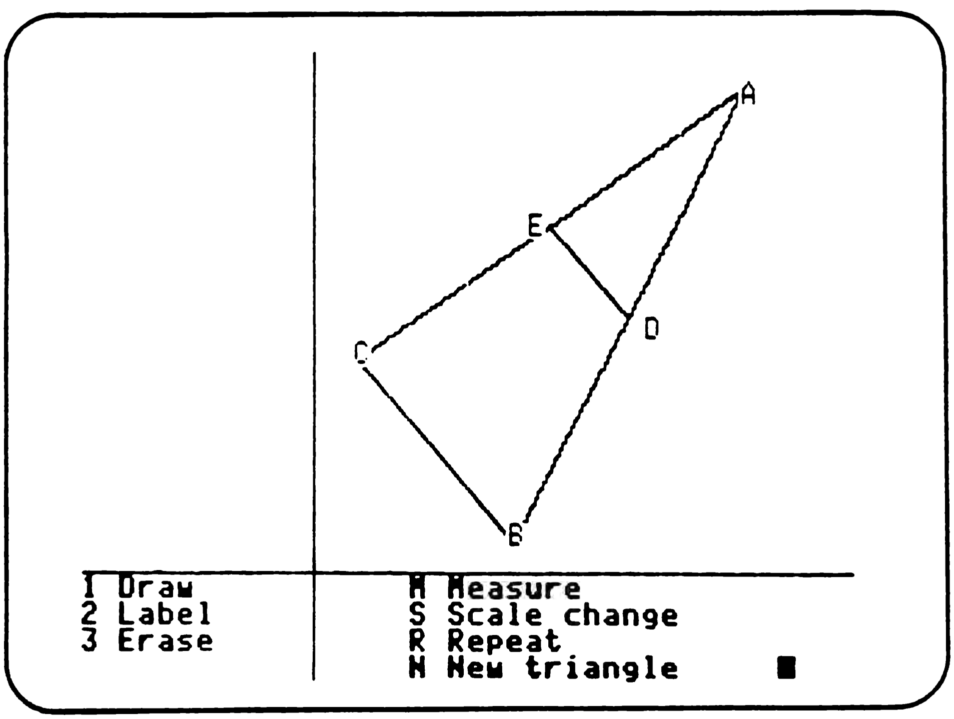
From midpoint of segment:

Enter the name of the first segment (AB) and the message will now read:

From midpoint of segment: AB
To midpoint of:

Enter the name of the second segment (BC) and the SUPPOSER will draw the midsegment, connecting the midpoints of segments AB and BC, and will label it.

Here, for example, is triangle ABC with midsegment DE drawn:



0 Extension

Choice 0 under Draw is Extension. This option allows you to extend a line segment from either end. The SUPPOSER will ask you to specify which line segment you wish to extend, from which end you wish to extend it, and then to specify the length of the extension you wish to draw in terms of 1) a segment or the unit length, or 2) to intersect segments on the screen.

Suppose you want to extend side AB from point B, with the extension equal in length to side AC. After entering the number 0, the message on the screen will read:

Extend segment:

Enter the name of the segment you want to extend (AB) and the message will now read:

Extend segment: AB
From point:

Enter the name of the point from which you want the segment to extend (B) and the message will change to:

Length defined:
1 by (segment or unit) * constant
2 to intersect segment (s)

Enter the number 1 to define the length of the extension by a length equal to segment AC. Now enter AC for segment or unit length. The message will read:

Length defined:
(segment or unit) * constant =
AC *

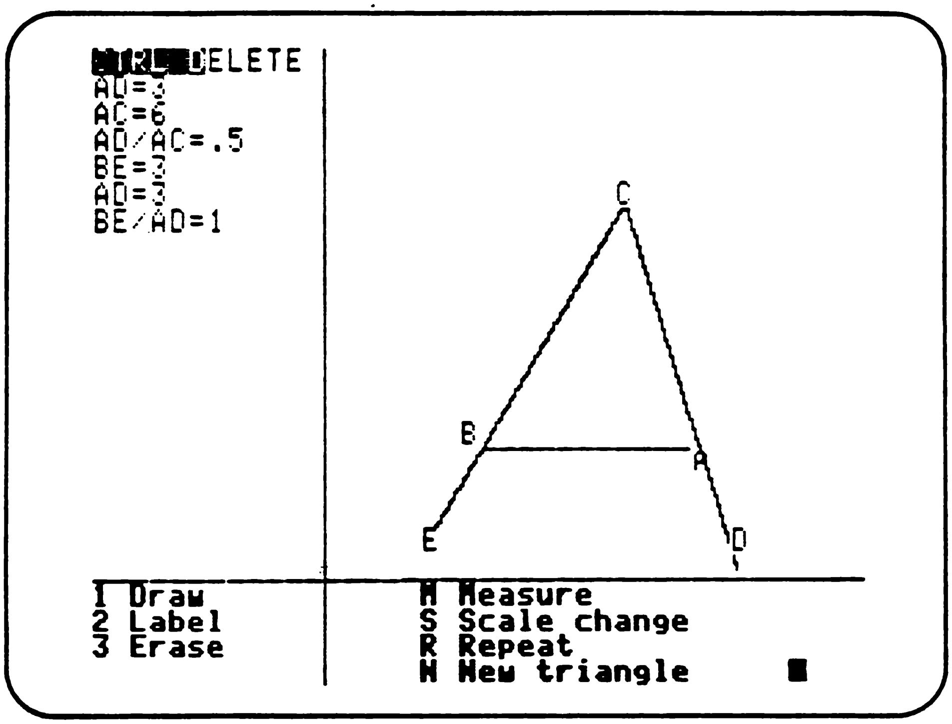
Now enter 1 for the constant and press RETURN. The SUPPOSER will extend the segment and label it.

If instead you want to define the length of the extension in terms of some multiple of the unit length u, enter u for segment length and then for constant, enter the multiplier that you want to use. Press RETURN and the SUPPOSER will draw the extension and label it.

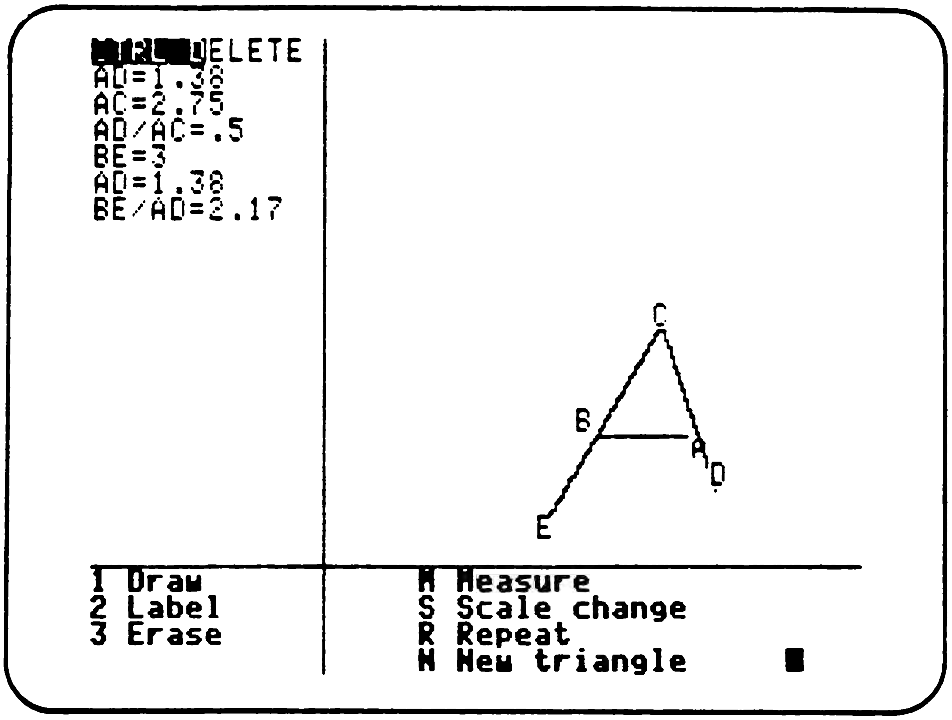
To illustrate the distinction between these two ways of specifying the length of the extension of a line segment, consider the following example and figures. Triangle ABC is an isosceles triangle with side lengths AB = AC = 6. Suppose we extend AC through A. Let the length of this extension be AC * .5. Suppose, in addition, we extend BC through B. Let the length of this extension be 3 units long.

The length of the extended line segment drawn through A is defined in terms of the length of the segment AC. The length of the extended line segment drawn through B is defined in terms of a certain number of screen units. Even though the lengths of the two extended line segments have the same numerical value, when we repeat the construction on a different triangle, or if we rescale the drawing, the two extensions will behave differently.

Here is the extension constructed in isosceles triangle ABC:



In this figure, the extension has been repeated on another triangle. Note the difference in the length of the extension drawn through A (defined in terms of AC) when the construction is repeated.



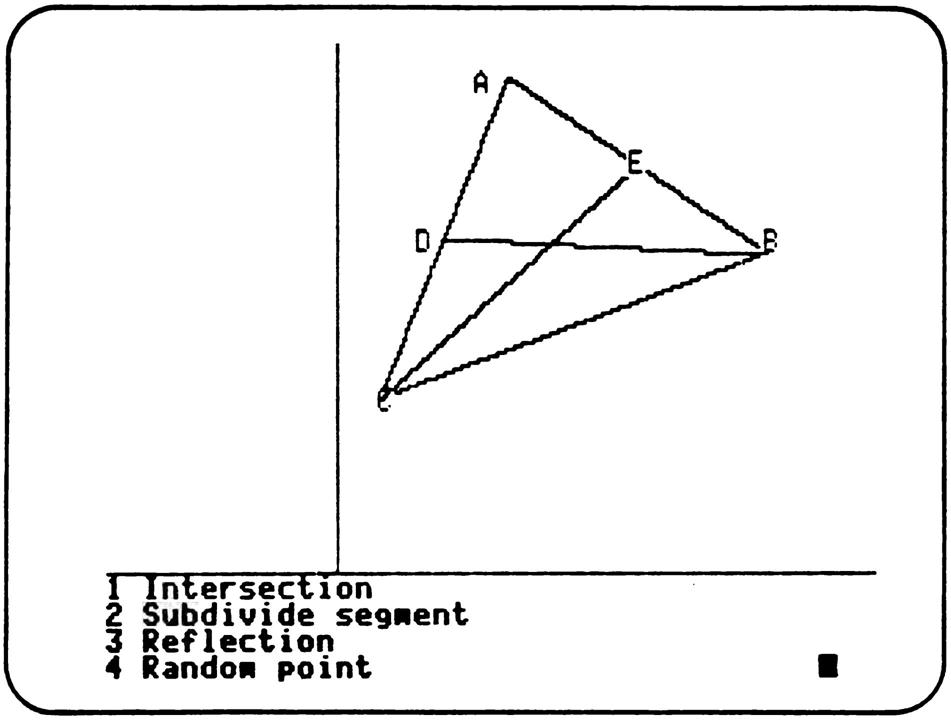
The second way to specify the length of an extension, to intersect segment(s), allows you to draw the extension until it intersects the segment or segments you name. When the message on the screen reads

> Length defined by:
> 1 (segment or unit) * constant
> 2 to intersect segment(s)

enter the number 2 and then the name of the segment that you want the extension to intersect. Press RETURN and the SUPPOSER will draw the extension to intersect that segment. If you want the extension to intersect two segments, enter the name of the first segment, then the name of the second segment. The SUPPOSER will draw the extension to intersect the segments that you have named.

LABEL

Choosing this option enables you to label various points on your triangle or construction. After pressing L you will see the following submenu screen:



Label

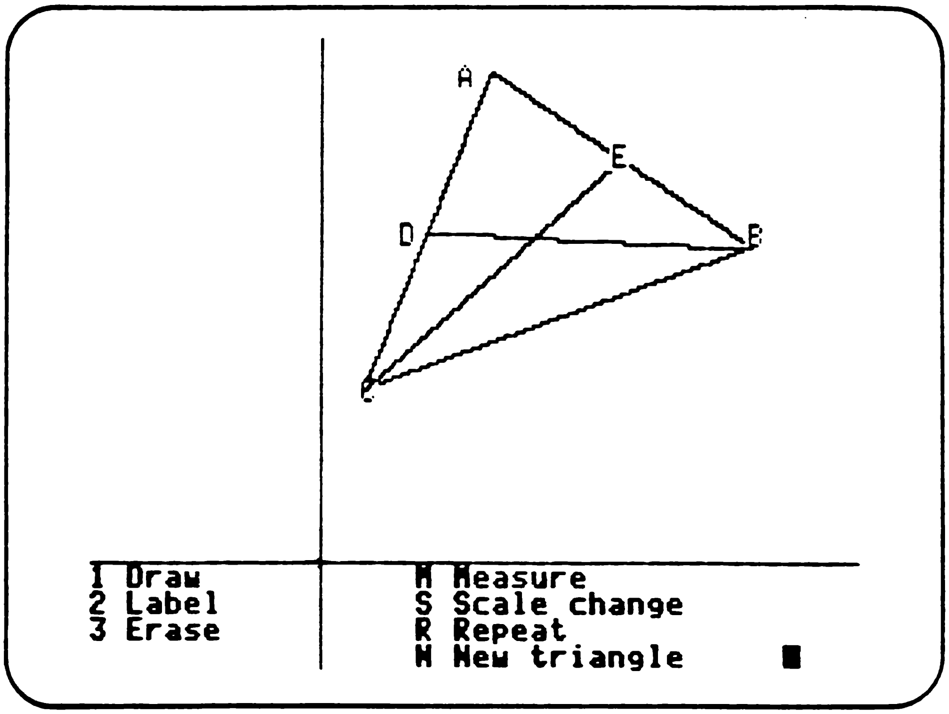
1 Intersection

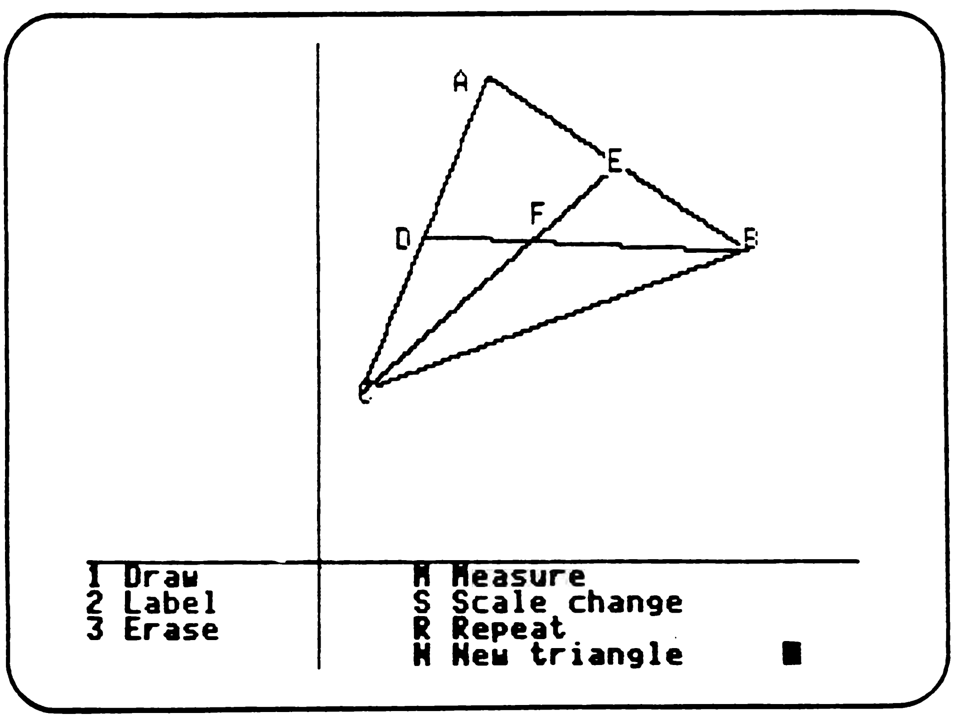
Choosing Intersection allows you to label the intersection of two line segments. The SUPPOSER will ask you to specify the lines whose intersection you want to label.

> Suppose you want to label the intersection of lines AB and DE. After entering the number 1, the message on the screen will read:
>
> Intersection of line:
>
> Enter the two letters that label the one line segment (AB) and the message will now read:
>
> Intersection of line: AB
> With line:
>
> Now enter the two letters that label the other line (DE). The SUPPOSER will label the intersection and return to the MAIN MENU.

There is no way to label the intersection of a circle and a line segment or the intersection of two circles.

Here, for example, is triangle ABC with medians drawn to sides AC and AB. The intersection of the medians has been labeled.





2 Subdivide segment

This choice allows you to subdivide any segment into any number of subdivisions. The SUPPOSER will ask you to identify the name of the segment to be subdivided and the number of subdivisions that you desire. (The number of subdivisions may not exceed 8.)

Suppose you want to divide segment AC into five sections. After entering the number 2, the message on the screen will read:

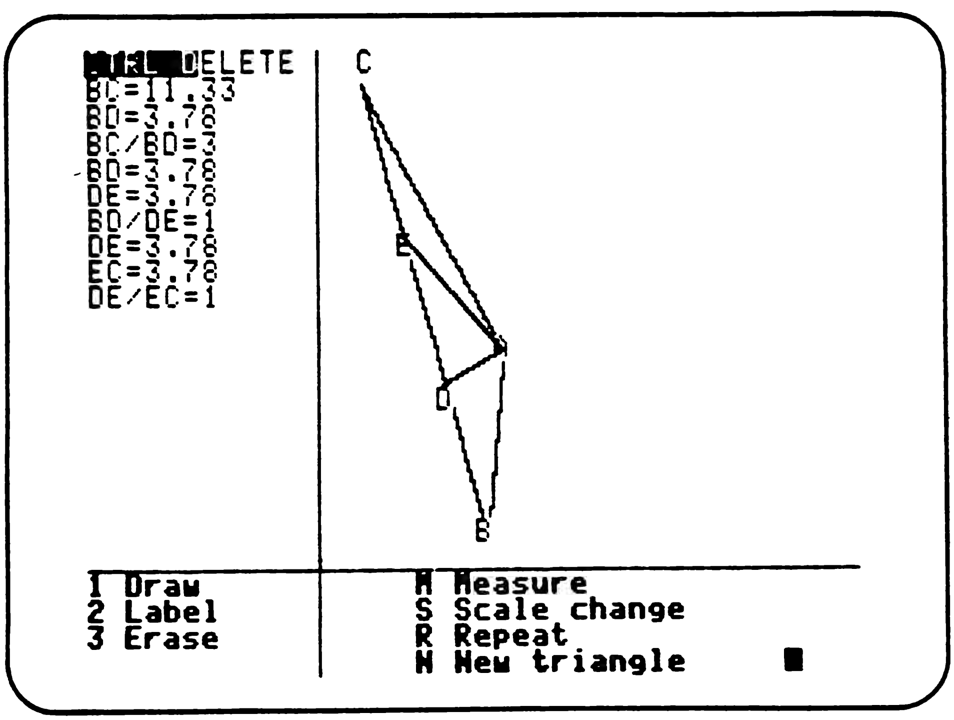
Subdivide segment:

Enter the name of the segment you wish to subdivide (AC). The message will now read:

Subdivide Segment: AC
Number of sections:

The number of sections can range from 2 to 8. Enter the the number of sections you desire (5) and the SUPPOSER will subdivide the segment, label the points of the subdivision, and return you to the MAIN MENU.

Here, for example, is an obtuse triangle ABC with segment BC divided into three equal-length segments. The line segments AD and AE are drawn in.



3 Reflection

This choice allows you to reflect either a point or a line segment in any line. The SUPPOSER will ask you to identify the point or line that you want reflected and in which line you want it reflected.

Suppose you want to reflect a point (A) in a line (BC) on the screen. Enter 3, and the message will read:

Reflection of:

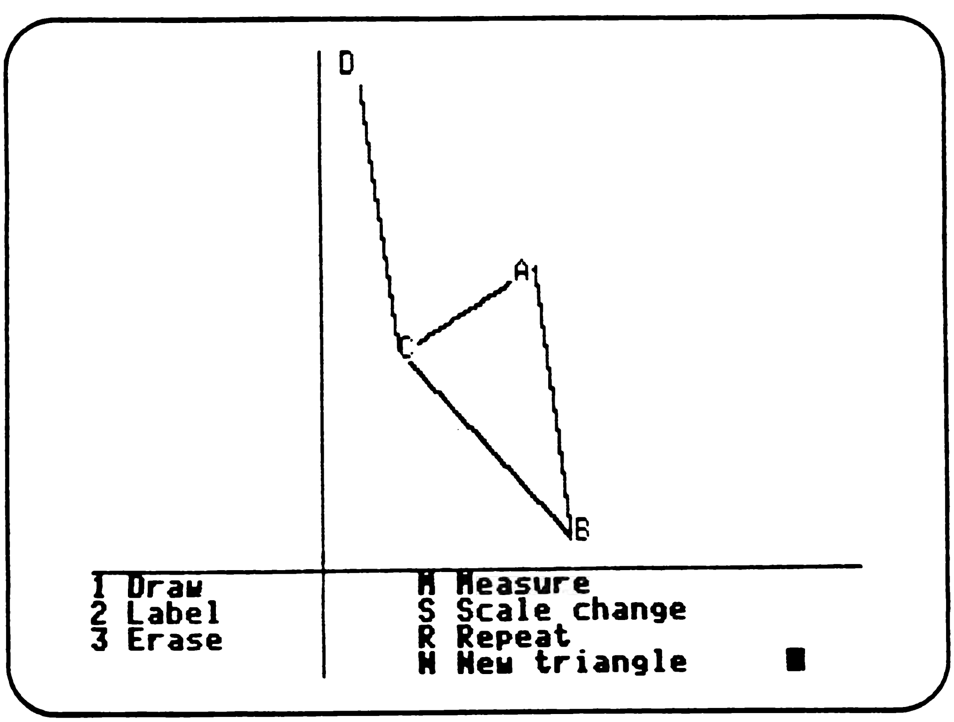
Enter the name of the point (A) and then press RETURN. After pressing RETURN, the message will read:

Reflection of: A   in:

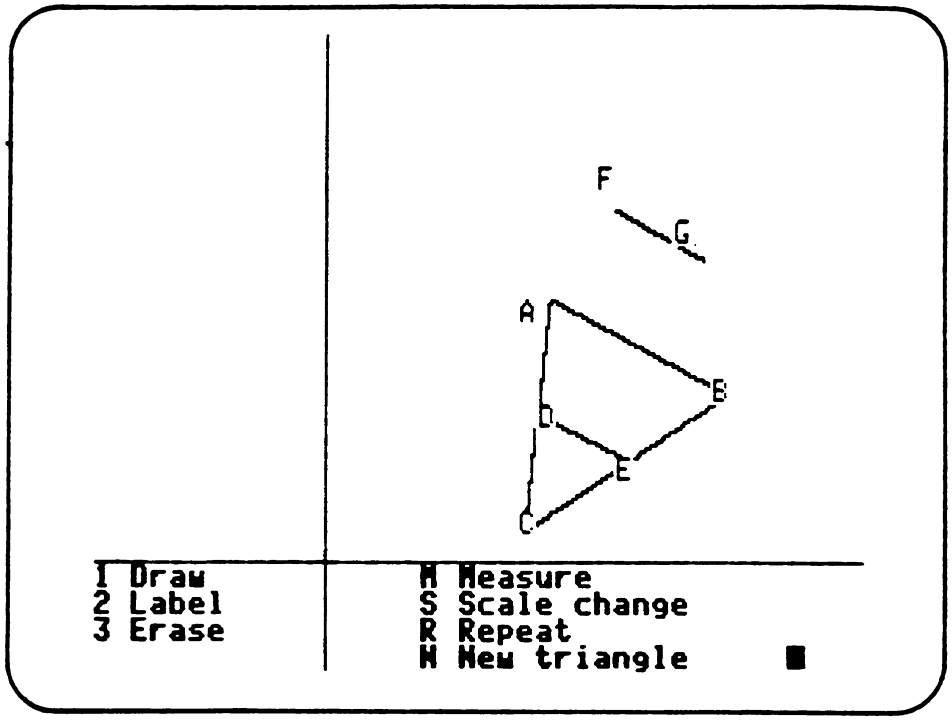
Now enter the name of the line segment in which you would like to reflect the point (BC). The SUPPOSER will locate and label the reflected point and return you to the MAIN MENU.

Suppose you want to reflect one line segment in another. When the above message appears, enter the name of the line segment you wish to reflect, and the letters that label the line segment in which the reflection is to be carried out. The SUPPOSER will locate and label the reflected line segment and return you to the MAIN MENU.

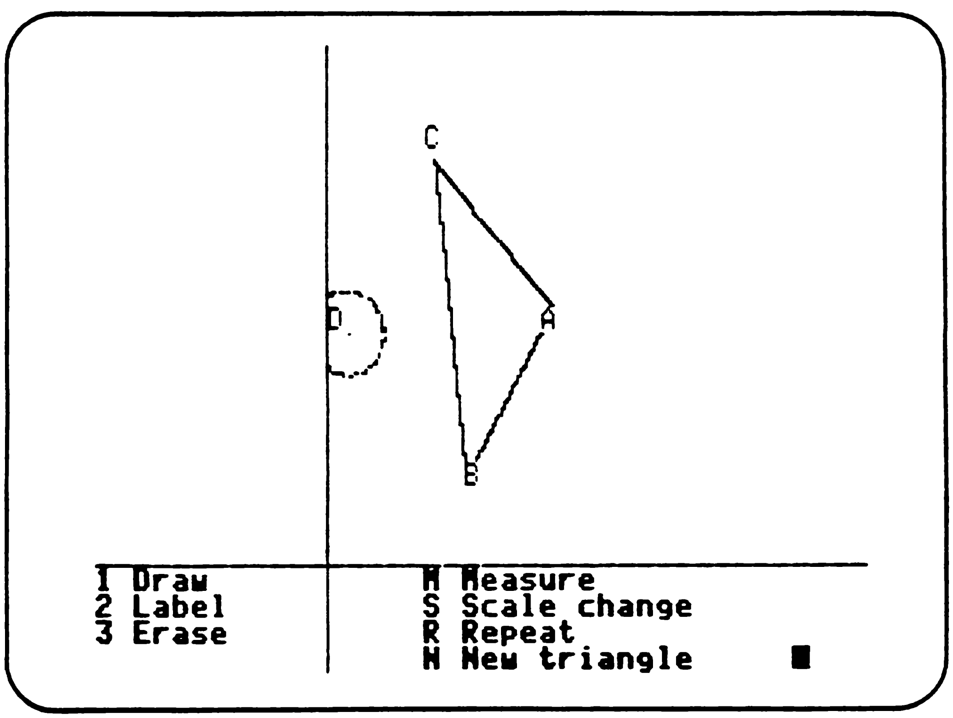
In this construction, the line segment CD is the reflection of line segment BC in line segment AC:



In this construction, the line segment FG is the reflection of line segment DE in line segment AB:



In this construction, the point D (with a circle drawn about it for clarity) is the reflection of point A in line segment BC:



4 Random point

This option allows you to place a point at random on a line segment, inside a triangle, or outside a triangle. The SUPPOSER will ask you to identify on which segment, or inside or outside of which triangle you wish a random point to be placed.

After entering the number 4, the message on the screen will read:

1 On segment:
2 Inside triangle:
3 Outside triangle:

Suppose you want to place a random point on a segment. Enter the number 1 and then enter the name of the segment on which you wish to place a point at random. The SUPPOSER will place the point on the segment and label it.

Suppose you want to place a random point inside a triangle. Enter the number 2 and then enter the name of the triangle in which you wish to place a point at random. The SUPPOSER will place the point inside the triangle and label it.

Suppose you want to place a random point outside a triangle. Enter the number 3 and then enter the name of the triangle outside of which you wish to place a point at random. The SUPPOSER will place the point outside the triangle and label it.

Erase

This option allows you to erase segments and to erase labels. The SUPPOSER will ask you which segment or label(s) you wish to erase.

After entering 3 for Erase from the MAIN MENU, the message on the screen will read:

1 Erase segment:
2 Erase label(s):

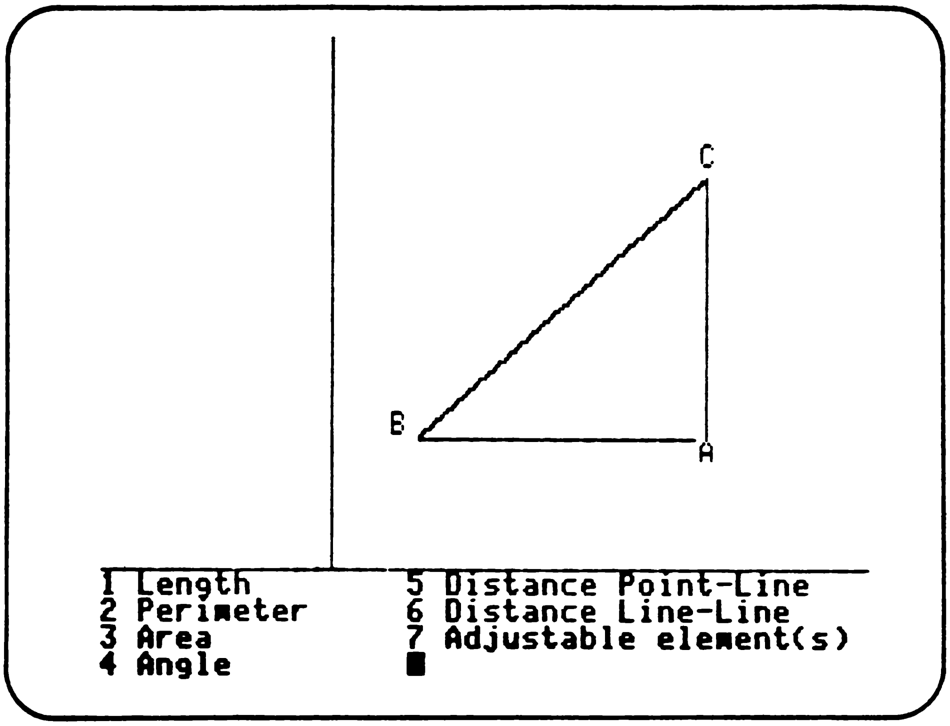
Suppose you wish to erase a segment. Enter the number 1 and then the name of the segment you wish to erase. The SUPPOSER will erase the segment and return you to the MAIN MENU.

Suppose you want to make a diagram less cluttered by erasing some of the labels. Enter the number 2, then the name(s) of the label(s) you wish to erase, and press RETURN. The SUPPOSER will erase the label from the screen and return you to the MAIN MENU.

Once you erase a segment or a label, it is still in the memory; thus you can still refer to it even though it does not appear on the screen. For example, if you erase the segment GC or the labels G and C, you can still draw a circle centered on G, measure the distance GC, or draw a parallel to CG.

MEASURE

The Measure option allows you to measure various components of your construction. Pressing M will result in the following submenu screen:



NOTE: Due to the graphic limitations of your computer, measurements will not always be precise. For example, you may get a measurement of 8.01 for the side of a triangle that you had inputed as 8. However, the point of being able to measure is not so much to get exact figures, as to enable students to be able to make conjectures, and then try to support their ideas.

1 Length

Choice 1 under Measure is Length; this option allows you to measure the length of any segment on the screen. You may also measure the distance between labeled points on the screen that have no line segment drawn between them. You can measure the sum, difference, product, and ratio of any two lengths and square any length as well. The SUPPOSER will ask you to identify the name of the segment whose length you wish to measure and will display the value of the length at the bottom of the screen and in the Data Column on the left of the screen. (The Data Column will display all your measurements, will scroll when it is full, and will clear when you create a new triangle. You may also clear the column at any time you are measuring by entering CONTROL D.) All measurements are in terms of the unit length u, displayed in the upper right-hand corner of the screen.

Suppose you want to measure the length of side AB in triangle ABC. After entering the number 1, the message on the screen will read:

Length of segment:

Enter the name of the segment (AB) and press RETURN. The SUPPOSER will measure the segment and display the value at the bottom of the screen and in the Data Column. The same procedure can be used to measure the distance between any two labeled points on the screen that are not connected by a segment.

You now have two options: To continue to measure segment lengths, press the SPACE BAR; to return to the MAIN MENU, press RETURN.

Suppose you want to measure the ratio of two sides of a triangle, for example, the ratio of side AB to side AC in a triangle where AB=5, AC=6, BC=7. After entering the number 1, the screen will read:

Length of segment:

Enter the name of the first segment (AB) and the screen will read:

Length of segment: AB

+,-,*,/,^ or RETURN

Now press / (located on the same key as "?"). The screen will read

    Length of segment: AB = 5   /
    Length of segment:

and the value (AB=5) will appear in the Data Column. Now enter the name of the second segment (AC) and press RETURN. The SUPPOSER will display the value of the second measure (AC=6) and give you the ratio of the two segments (AB/AC=.83) at the bottom of the screen and in the Data Column. Now you can either continue to measure lengths by pressing the SPACE BAR, or return to the MAIN MENU by pressing RETURN.

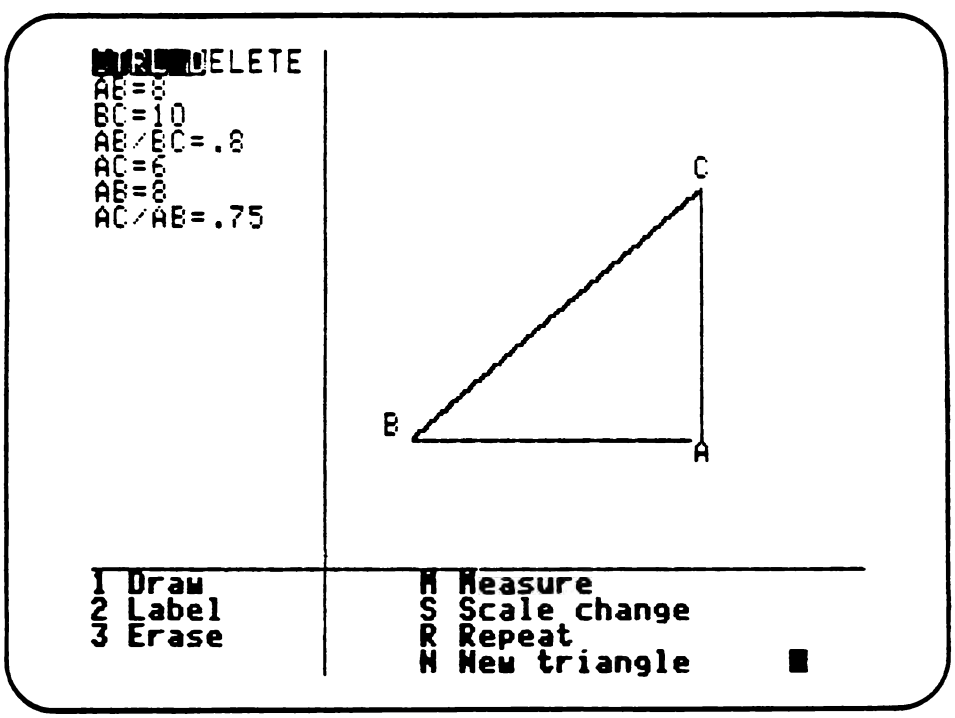
To measure the sum of two lengths, follow the same procedure, but press the + key rather than / after entering the name of the first segment.

To measure the difference between two lengths, follow the same procedure, but press the - key rather than the / after entering the name of the first segment.

To measure the product of two lengths, follow the same procedure, but press the * key rather than the / after entering the name of the first segment.

To square the measure of any length, press the ^ key after you enter the name of the segment.

Here, for example, are the results of some length measurements on a triangle the length of whose sides is 6:8:10:



2 Perimeter

Choice 2 under Measure is Perimeter; this option allows you to measure the perimeter of any triangle on the screen. You may also measure the perimeter of a triangle that is defined by three labeled points but is not drawn on the screen. You can measure the sum, difference, product, and ratio of any two perimeters and square the measure of a perimeter as well. The SUPPOSER will ask you to specify the name of the triangle whose perimeter you wish to measure and will display the value of the perimeter at the bottom of the screen and in the Data Column on the left of the screen. (The Data Column will display all your measurements, will scroll when it is full, and will clear when you create a new triangle. You may also clear the column at any time you are measuring by entering CONTROL D.) All measurements are in terms of the unit length u, displayed in the upper right-hand corner of the screen.

Suppose you want to measure the perimeter of triangle ABC. After entering the number 2, the message will read:

Perimeter of triangle:

Now enter the name of the triangle (ABC) whose perimeter you wish to measure and press RETURN. The SUPPOSER will measure the perimeter and display the value at the bottom of the screen and in the Data Column. The same procedure can be used to measure the perimeter of a triangle defined by three points on the screen, but not drawn.

You now have two options: To continue to measure perimeters, press the SPACE BAR; to return to the MAIN MENU, press RETURN.

Suppose you want to measure the ratio of the perimeters of two triangles, ADC and ABD, created by constructing an altitude from vertex A in triangle ABC, where AB=5, AC=6, and BC=7. After entering the number 2, the message on the screen will read:

Perimeter of triangle:

Enter the name of the first triangle (ADC) and the message will read:

Perimeter of triangle: ADC

+, -, *, /, ^   or   RETURN

Now press / (located on the same key as "?"). The screen will now read

Perimeter of triangle: ADC = 14.4 /
Perimeter of triangle:

and the value (P:ADC= 14.4) will appear in the Data Column. Now enter the name of the second triangle (ABD) and press RETURN. The SUPPOSER will display the value of the second perimeter (ABD=11.9) and give you the ratio of the two perimeters (ADC/ABD=1.21) at the bottom of the screen and in the Data Column. Now you can either continue to measure perimeters by pressing the SPACE BAR, or return to the MAIN MENU by pressing RETURN.

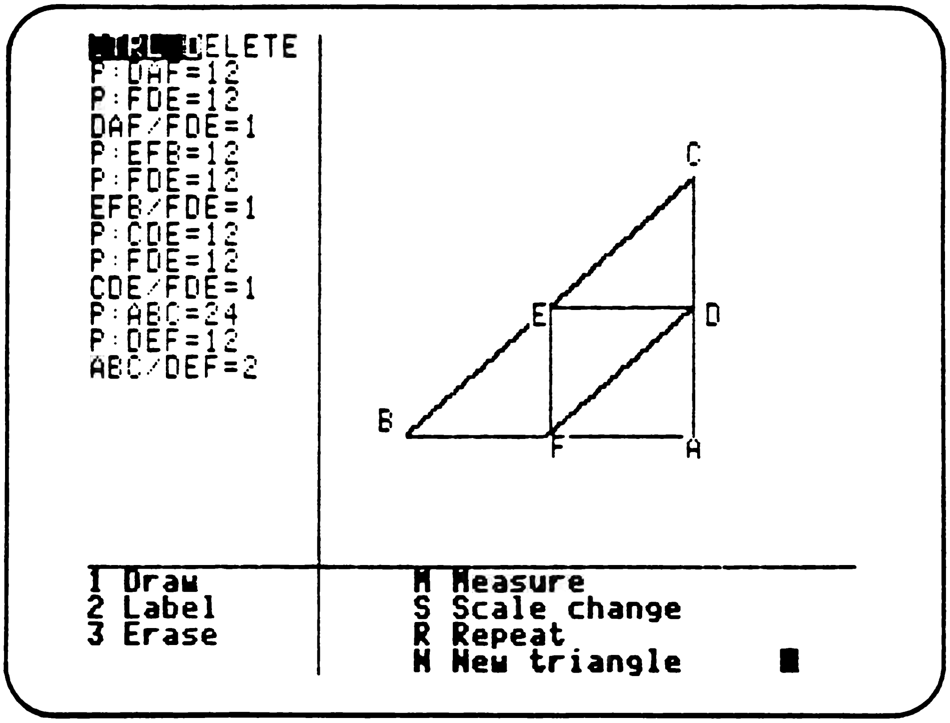
To measure the sum of two perimeters, follow the same procedure, but press the + key rather than / after entering the name of the first perimeter.

To measure the difference between two perimeters, follow the same procedure, but press the - key rather than the / after entering the name of the first perimeter.

To measure the product of two perimeters, follow the same procedure, but press the * key rather than the / after entering the name of the first perimeter.

To square the measure of a perimeter, press the ^ key after entering the name of the triangle.

Here are the results of some perimeter measurements on a triangle and all of its midsegments:



3 Area

Choice 3 under Measure is Area; this option allows you to measure the area of any triangle on the screen. You may also measure the area of a triangle that is defined by three labeled points but is not drawn on the screen. You can measure the sum, difference, product, and ratio of any two areas, and square an area as well. The SUPPOSER will ask you to name the triangle whose area you wish to measure and will display the value of the area at the bottom of the screen and in the Data Column on the left of the screen. (The Data Column will display all your measurements, will scroll when it is full, and will clear when you create a new triangle. You may also clear the column at any time you are measuring by entering CONTROL D.) All measurements are in terms of the unit area u, displayed in the upper right-hand corner of the screen.

Suppose you want to measure the area of triangle ABC. After entering the number 3, the message on the screen will read:

Area of triangle:

Now enter the name of the triangle whose area you want to measure (ABC) and press RETURN. The SUPPOSER will measure the area and display the value at the bottom of the screen and in the Data Column. The same procedure can be used to measure the area of a triangle defined by three points on the screen, but not drawn.

You now have two options: To continue to measure areas, press the SPACE BAR; to return to the MAIN MENU, press RETURN.

Suppose you want to measure the ratio of the areas of two triangles, ADC and ABD, created by constructing an altitude from vertex A in triangle ABC, where AB=5, AC=6, and BC=7. After entering the number 3, the screen will read:

Area of triangle:

Enter the name of the first triangle (ADC) and the screen will read:

Area of triangle: ADC

+,-,*,/,^ or RETURN

Now press / (located on the same key as "?"). The screen will read

Area of triangle:ADC = 8.99 /
Area of triangle:

and the value (A:ADC=8.99) will appear in the Data Column. Now enter the name of the second triangle (ABD) and press RETURN. The SUPPOSER will display the value of the second area (ABD=5.69) and give you the ratio of the two areas (ADC/ABD=1.58) at the bottom of the screen and in the Data Column. Now you can either continue to measure areas by pressing the SPACE BAR, or return to the MAIN MENU by pressing RETURN.

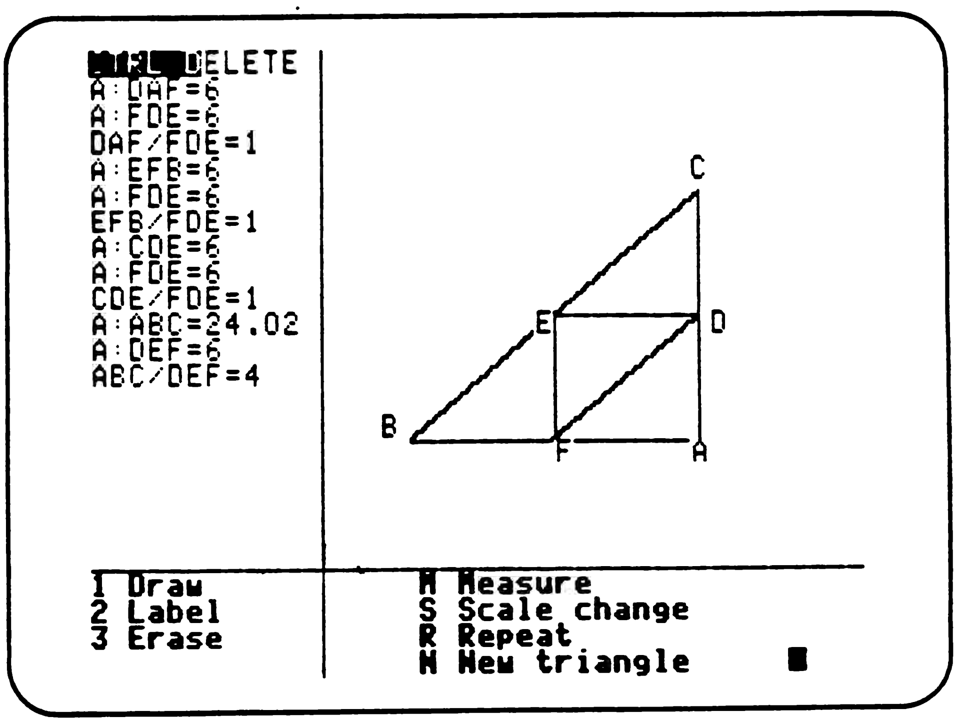
To measure the sum of two areas, follow the same procedure, but press the + key rather than / after entering the name of the first area.

To measure the difference between two areas, follow the same procedure, but press the - key rather than the / after entering the name of the first area.

To measure the product of two areas, follow the same procedure, but press the * key rather than the / after entering the name of the first area.

To square an area, press the ^ key after entering the name of the triangle.

Here are the results of some area measurements on the set of triangles formed by constructing all the midsegments of a triangle:



4 Angle

Choice 4 under Measure is Angle; this option allows you to measure any angle on the screen. You may also measure an angle that is defined by three labeled points but is not drawn on the screen. You can measure the sum, difference, product, and ratio of any two angles and square the measure of an angle as well. The SUPPOSER will ask you to name the angle you wish to measure and will display the value of the angle at the bottom of the screen and in the Data Column on the left of the screen. (The Data Column will display all your measurements, will scroll when it is full, and will clear when you create a new triangle. You may also clear the column at any time you are measuring by entering CONTROL D.)

Suppose you want to measure the angle ABC in triangle ABC where AB=5, AC=6, BC=7. After entering the number 4, the message will read:

Angle name (3 letters):

Now enter the name of the angle you want to measure (ABC) and press RETURN. The SUPPOSER will measure the angle and display the value (57) at the bottom of the screen and in the Data Column. The same procedure can be used to measure an angle defined by three points on the screen, but not drawn.

You now have two options: To continue to measure angles, press the SPACE BAR; to return to the MAIN MENU, press RETURN.

Suppose you want to measure the ratio of two angles, angle ABC and angle CAB, in triangle ABC cited above. After entering the number 4, the screen will read:

Angle name (3 letters):

Enter the letters that label the first angle (ABC) and the screen will read:

Angle name (3 letters): ABC

+,-,*,/,^ or RETURN

Now press / (located on the same key as "?"). The screen will read

Angle name (3 letters): ABC = 57 /
Angle name (3 letters):

and the value (ABC= 57) will appear in the Data Column. Now enter the name of the second angle

(CAB) and press RETURN. The SUPPOSER will display the value of the second angle (CAB=78) and give you the ratio of the two angles (ABC/CAB=.73) at the bottom of the screen and in the Data Column. Now you can either continue to measure angles by pressing the SPACE BAR, or return to the MAIN MENU by pressing RETURN.

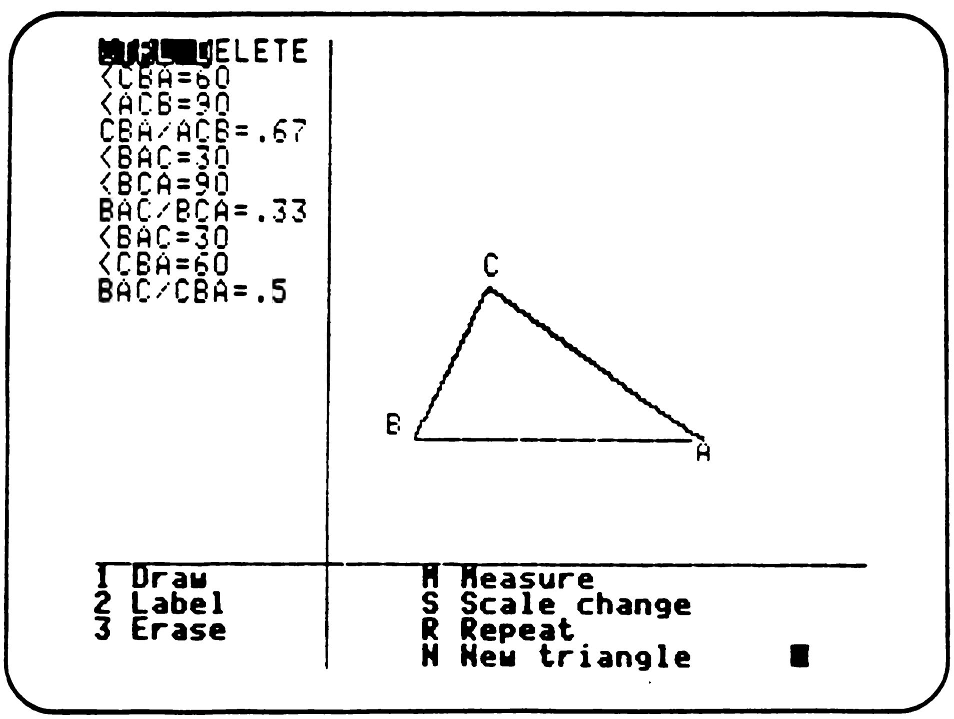
To measure the sum of two angles, follow the same procedure, but press the + key rather than / after entering the name of the first angle.

To measure the difference between two angles, follow the same procedure, but press the - key rather than the / after entering the name of the first angle.

To measure the product of two angles, follow the same procedure, but press the * key rather than the / after entering the name of the first angle.

To square the distance, press the ^ key after entering the names of the point and the segment that define the distance.

Here, for example, are the results of some angle measurements in a familiar triangle:



5 Distance point-line

Option 5 under Measure is Distance Point-Line. This option allows you to measure the perpendicular distance from any labeled point on the screen to any line segment on the screen. You may also measure the distance from a point to a line segment defined by two points but not drawn. You can measure the sum, difference, product, and ratio of such distances and square the distance as well. The SUPPOSER will ask you to name the point and the line segment that define the distance you wish to measure. The Data Column will display all your measurements, will scroll when it is full, and will clear when you create a new triangle. You may also clear the column at any time you are measuring by entering CONTROL D. All measurements are in terms of the unit length u, displayed in the upper right-hand corner of the screen.

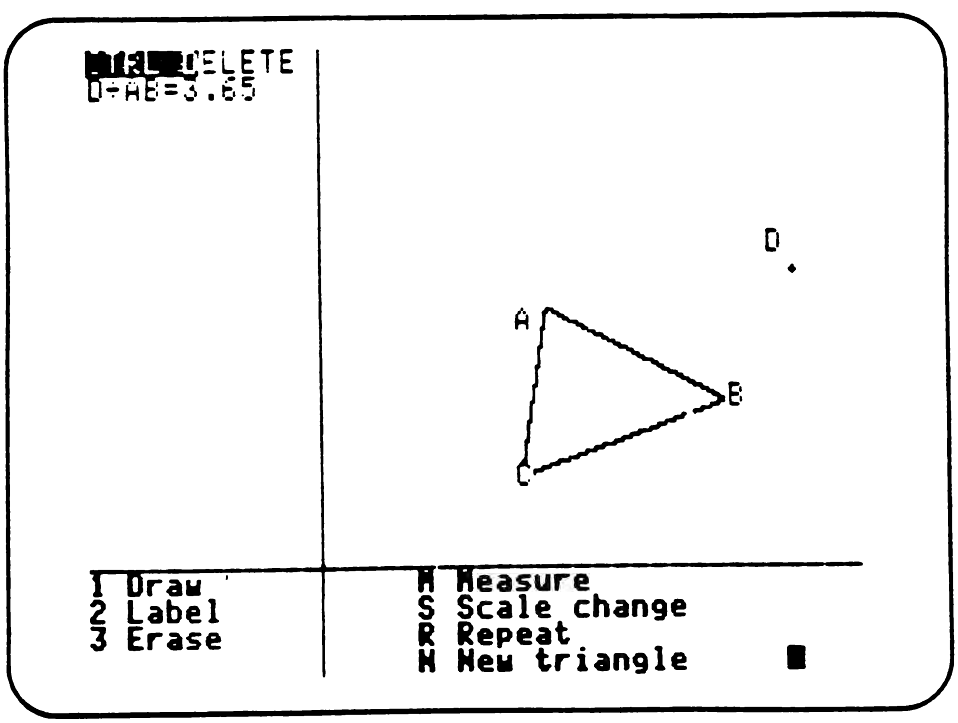
Suppose you want to measure the perpendicular distance from a random point D, located outside of triangle ABC, to segment AB. After entering the number 5, the message will read:

Distance of point:

Enter the name of the point (D) and the message will now read:

Distance of point: D to line:

Now enter the name of the segment (AB) and press RETURN. The SUPPOSER will measure the perpendicular distance and display the value at the bottom of the screen and in the Data Column.



You now have two options: To continue to measure point-line distances, press the SPACE BAR; to return to the MAIN MENU, press RETURN.

Suppose you want to measure the ratio of two such distances, the ratio of the perpendicular distance of point A to line BC to the perpendicular distance of point D to line BE, where DE is a midsegment drawn between sides AB and BC in triangle ABC. After entering the number 5, the message will read:

Distance of point:

Enter the name of the first point (A) and the message will now read:

Distance of point: A   to line:

Enter the name of the first segment (BC) and the message will read:

Distance of point: A   to line: BC

+, -, *, /   or   RETURN

Now press / (located on the same key as "?") and the message will read:

Distance of point: A   to line: BC = 8.7   /
Distance of point:

displaying the value of the first perpendicular distance at the bottom of the screen and in the Data Column. Now enter the name of the point (D) and the name of the segment (BE) of the second distance that you want to measure, and press RETURN. The SUPPOSER will display the value of the second distance and give you the ratio of the distances at the bottom of the screen and in the Data Column. Now you can either continue to measure point-line distances by pressing the SPACE BAR, or return to the MAIN MENU by pressing RETURN.

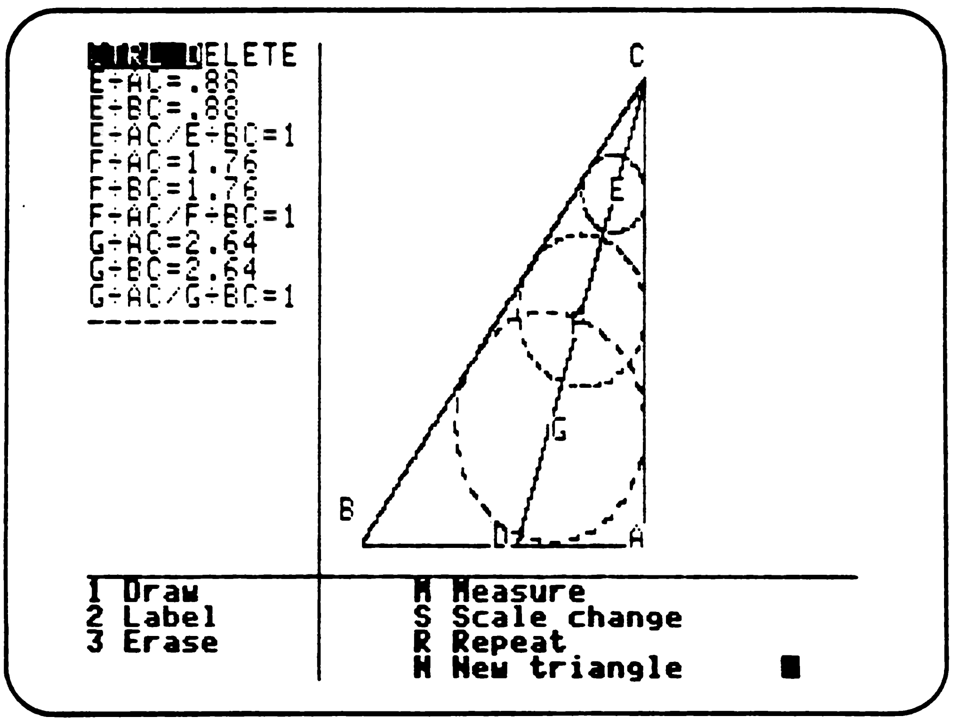
To measure the sum of two such distances, follow the same procedure, but press the + key rather than / after entering the names of the point and segment for the first distance.

To measure the difference between two such distances, follow the same procedure, but press the - key rather than / after entering the names of the point and segment for the first distance.

To measure the product of two such distances, follow the same procedure, but press the * key rather than / after entering the names of the point and segment for the first distance.

To square the distance, press the ^ key after entering the names of the point and the segment that define the distance.

Here are the results of a set of measurements of point-line distances.



6 Distance line-line

Option 6 under Measure is Distance Line-Line. This option allows you to measure the perpendicular distance between any two parallel line segments on the screen. You may also measure the distance between parallel line segments that are labeled but not drawn. You can measure the sum, difference, product, and ratio of such distances and to square such a distance as well. The SUPPOSER will ask you to name the letters that label the two segments (lines) whose distance you wish to measure. All measurements are in terms of the unit length u, displayed in the upper right-hand corner of the screen.

If you ask the SUPPOSER to measure the distance between two non-parallel or intersecting line segments, the message will read

Intersecting lines

since the perpendicular distance between non-parallel or intersecting lines cannot be defined.

The Data Column will display all your measurements, will scroll when it is full, and will clear when you create a new triangle. You may also clear the column at any time you are measuring by entering CONTROL D.

Suppose you want to measure the perpendicular distance from segment DE to side BC, where DE is a parallel drawn through point A and parallel to side BC. After entering the number 6, the message will read:

Distance of line:

Enter the name of the first segment (DE) and the message will now read:

Distance of line: DE to line:

Now enter the name of the second segment (BC) and press RETURN. The SUPPOSER will measure the perpendicular distance and display the value at the bottom of the screen and in the Data Column.

You now have two options: To continue to measure line-line distances press the SPACE BAR; to return to the MAIN MENU, press RETURN.

Suppose you want to measure the ratio of two such distances, the ratio of the perpendicular distance of segment DE to line BC to the perpendicular distance of

line BC to line GF, where a midsegment labeled GF drawn from side AC to side BA has been added to the construction described above. After entering the number 6, the message will read:

Distance of line:

Enter the name of the first segment (DE) and the message will now read:

Distance of line: DE to line

Enter the name of the second segment (BC) and the message will read:

Distance of line: DE to line: BC

+,-,*,/ or RETURN

Now press / (located on the same key as "?") and the message will read

Distance of line: DE to line: BC = 4.68 /
Distance of line:

displaying the value of the first perpendicular distance at the bottom of the screen and in the Data Column. Now enter the name of the segments that define the second perpendicular distance (BC and GF) and press RETURN. The SUPPOSER will display the value of the second distance and give you the ratio of the distances at the bottom of the screen and in the Data Column. Now you can either continue to measure line-line distances by pressing the SPACE BAR, or return to the MAIN MENU by pressing RETURN.

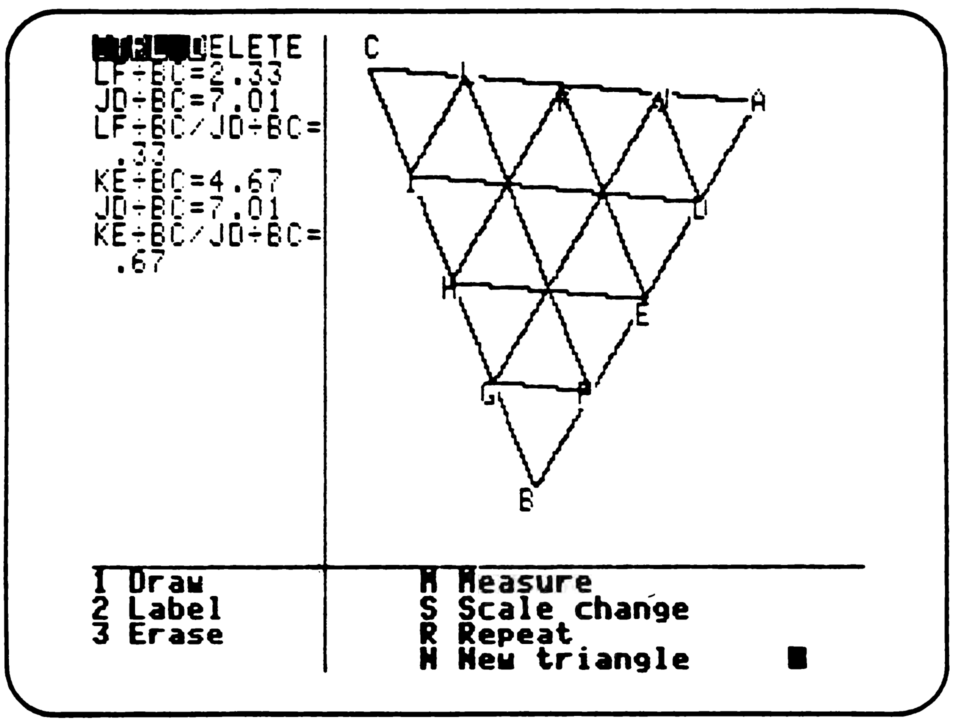
To measure the sum of two such distances, follow the same procedure, but press the + key rather than / after entering the names of the segments in the first distance.

To measure the difference between two such distances, follow the same procedure, but press the - key rather than / after entering the names of the segments in the first distance.

To measure the product of two such distances, follow the same procedure, but press the * key rather than / after entering the names of the segments in the first distance.

To square the measure of such a distance, press the ^ key after entering the names of the segments that define the distance.

Here are the results of a set of measurements on a set of similar triangles:



7 Adjustable element(s)

Choice 7 under Measure is Adjustable Elements; this option allows you to attach a label to a movable point, and to monitor the measure of up to three elements as you move the point about the screen. These elements may be any combination of lengths of segments, measures of angles, and distances from points to lines. Lengths and distances can be reported in terms of the standard unit length u, displayed in the upper right-hand corner of the screen, and/or in multiples of the length of any segment you choose.

The SUPPOSER will indicate the letter that will label the movable point and then ask you to indicate which lengths, angles, and/or distances you want to measure, and in what terms you want the measures reported. You can then move the point, in large and small steps, around the screen, taking measurements as often as you like. Movable points are labeled with inverse letters and all labels made after the use of a movable point appear as inverse letters.

> Suppose you have a triangle ABC (with the endpoint of a median labeled D) and you wish to place the movable point E such that triangle ABE, formed by the points A, B, and E, is equilateral--so that the lengths of AE and BE are equal to the length of AB. This means that you want to adjust the position of point E while you keep track of the lengths of segments AE and BE in terms of the length of segment AB.
>
> After entering the number 7, the message will read (assuming the most recent label used was D):
>
> Next point is E. Measured elements:
> L - Length
> A - Angle
> D - Distance
>
> Enter L to indicate that you want to measure a length and the cursor will position itself next to the word Length. Now enter the name of one of the lengths you wish to measure, in this case AE, and press RETURN. The cursor will return to the first line, next to the words Measured element. Now press L again to indicate that you wish to measure a second length. The cursor will position itself by Length again. Enter BE and press RETURN. Since in this example you do not wish to follow any other measures as you move the adjustable point, press RETURN a second time to indicate that you have identified all the elements you wish to measure.

The message at the bottom of the screen will now read:

Measure in terms of segment or unit:

Enter the name of a segment, or enter u for the standard unit length to specify the length in terms of which you want the lengths to be measured (AB) and press RETURN. The message at the bottom of the screen will change to

M - Measure S - Step (large) RETURN

and the movable point will appear in the lower right-hand corner of the screen. Now you can move the point and measure the adjustable elements.

To move the adjustable point, use the following keys:

	APPLE IIe or IIc	APPLE II+
LEFT	left arrow	left arrow
RIGHT	right arrow	right arrow
UP	up arrow	CTRL K
DOWN	down arrow	CTRL J

By pressing the S key, you can move back and forth between large steps and small steps and the message on the screen will indicate which size you are using. When you are using the small steps, the program will report the measurements with each step you take.

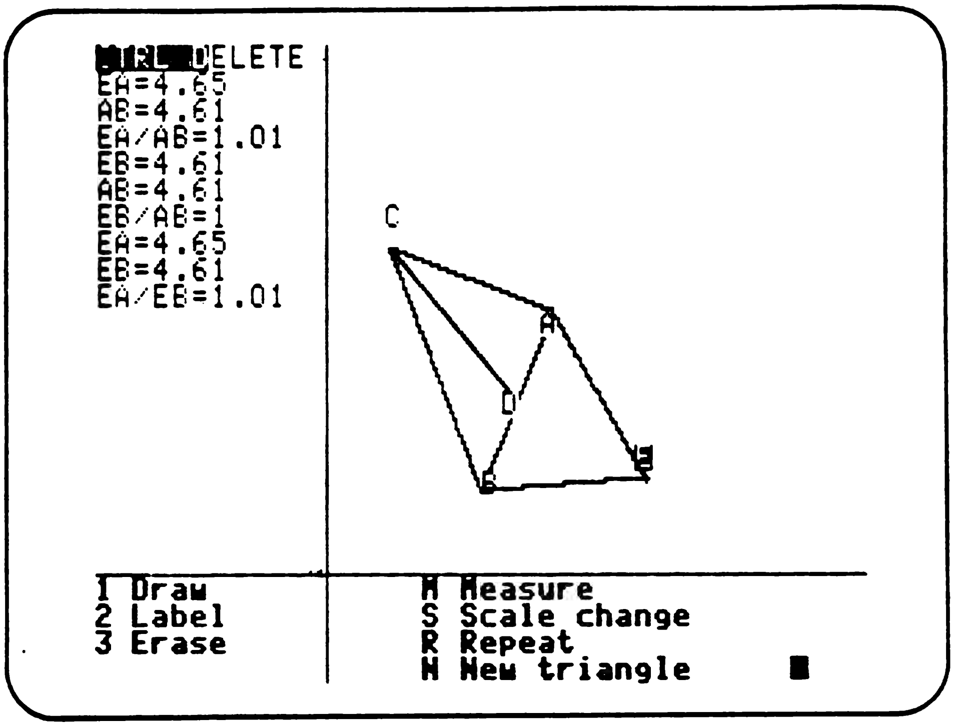
Now move the point so that ABE forms an equilateral triangle. When you have moved the point to a position that seems to be about where you would like it, press M (for Measure) to obtain the measure of the elements (in this case the length of segments AE and BE) you are monitoring. The program will report the measurements both in the standard unit length u and in terms of AB as indicated below:

M - Measure S - Steps (large) RETURN
AE = 4.68 = 1.1AB
BE = 3.83 = .9AB

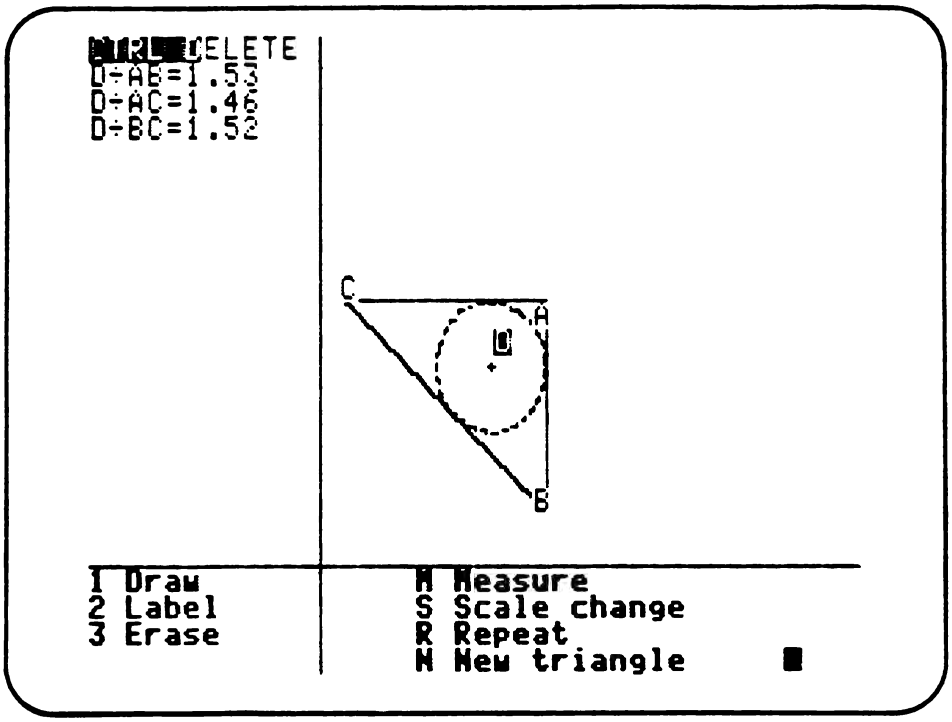
You can measure as often as you like in the course of moving the point.

When the point (E in this case) is located close to where you want it, press S to move the point in much smaller steps. Continue to adjust the position of the point until it is where you want it (in this case, so that AE = AB and BE = AB). Now press RETURN, and the point will be fixed and you will be returned to the MAIN MENU.

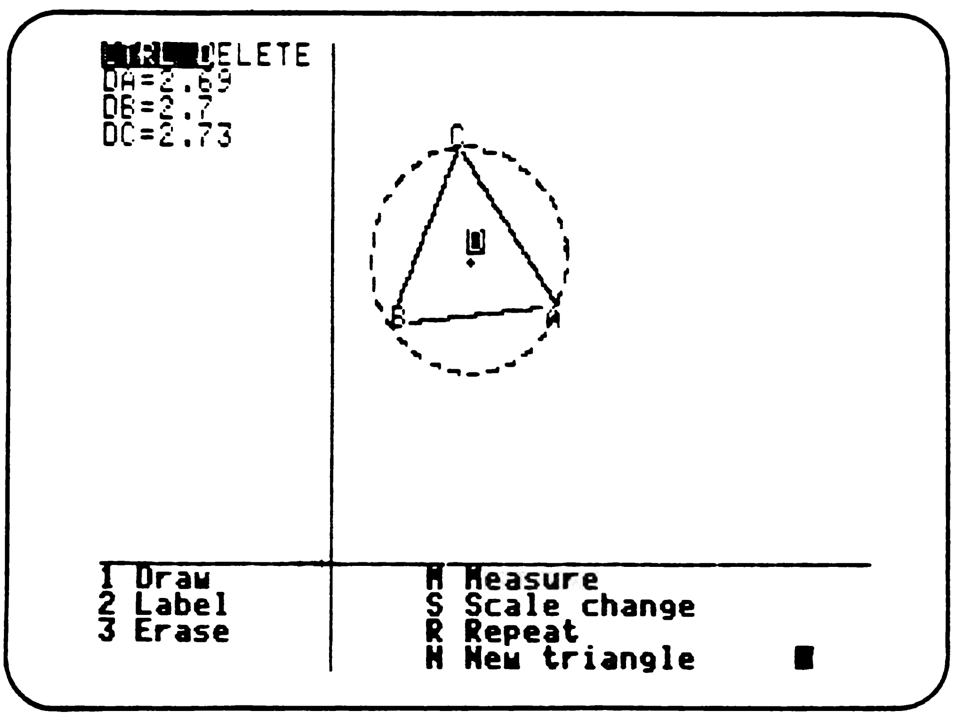
<u>Please note that placing the movable point in a precise location is difficult due to the limitations of the display.</u>



Suppose you wish to place the movable point D so that it is at the center of the circle that can be inscribed in triangle ABC--so that the distances from point D to sides AB, AC, and BC are equal. This means you need to monitor the distance from the point D to the line segments AB, AC, and BC.



Suppose you wish to place the movable point D so that it's at the center of the circle that circumscribes triangle ABC--so that the distances from D to the vertices A, B, and C are equal. This means you have to monitor the lengths of segments DA, DB, and DC.



Scale change

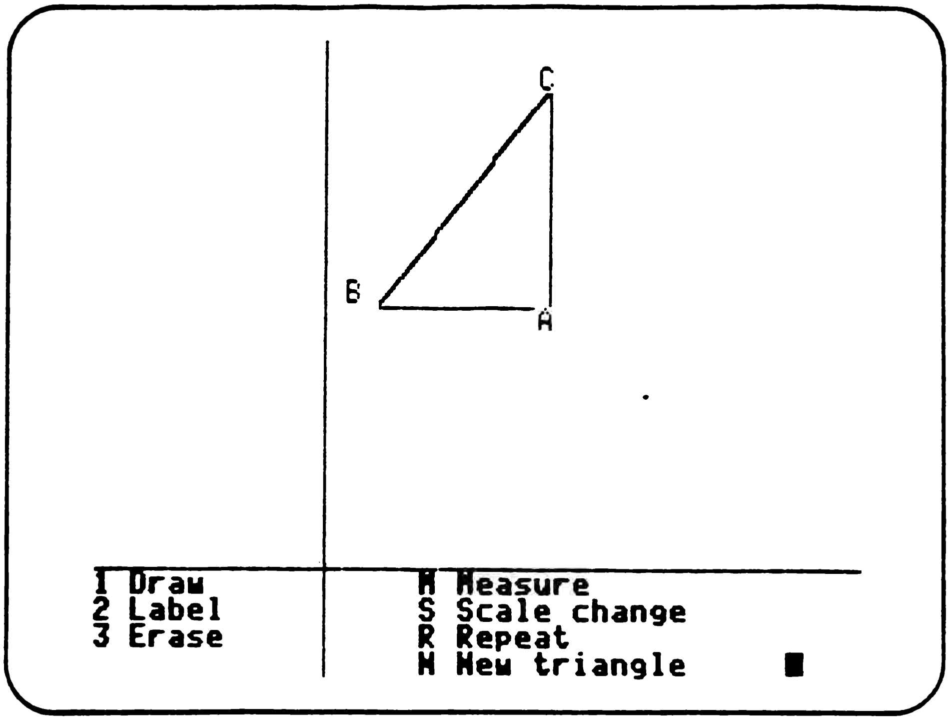
Any drawing on the screen can be drawn in one of two sizes. Choosing this option will cause the SUPPOSER to redraw the drawing on the screen in the OTHER size.

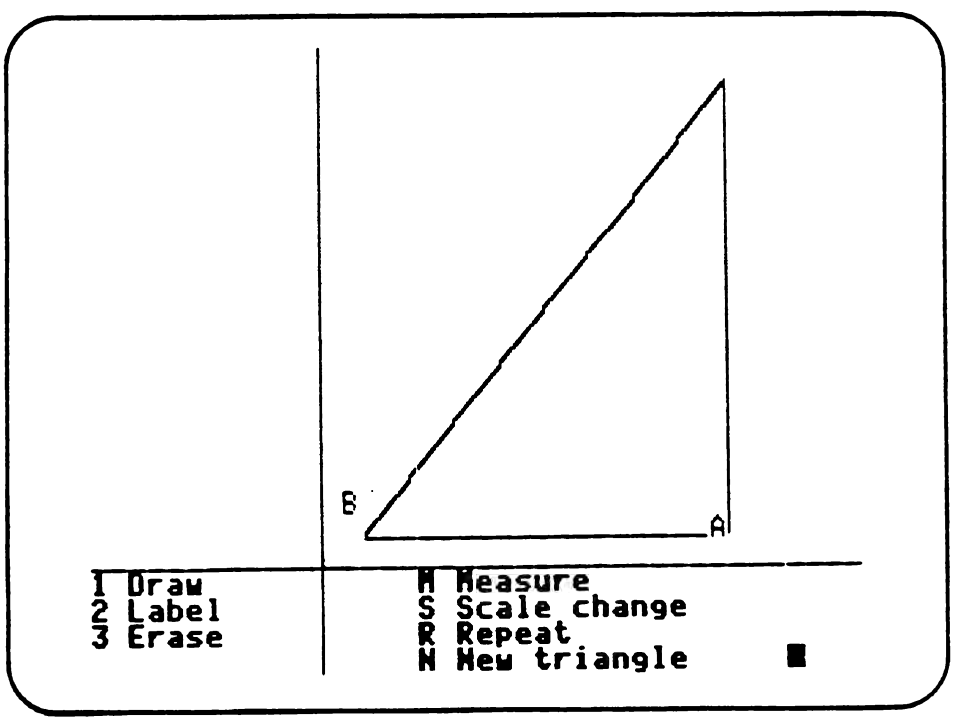
The option has another function as well. If a construction cannot be fully displayed on the screen and the beep sounds, try using the SCALE CHANGE option. In addition to changing the scale of the triangle, SCALE CHANGE checks the placement of the triangle on the screen. In most cases, the program can relocate the triangle so that the construction can be displayed.

> To carry out a scale change, enter the letter S and the SUPPOSER will rescale the triangle.

IMPORTANT NOTE: If you have used the adjustable element(s) option in MEASURE, the SUPPOSER will not rescale those parts of the construction that are connected to the movable point(s) (labeled with inverse letters) introduced into the construction. In addition, the SUPPOSER will not rescale any part of the construction made _after_ the first use of the _adjustable element(s)_ in the construction.

Here is triangle ABC, and the same triangle rescaled:





Repeat

The REPEAT option allows you to repeat the construction you have just made on a new triangle or on any one of the three most recent previous triangles. You can repeat a construction or series of constructions on as many triangles as you like.

To choose the REPEAT option in the MAIN MENU, press R and the message on the screen will read:

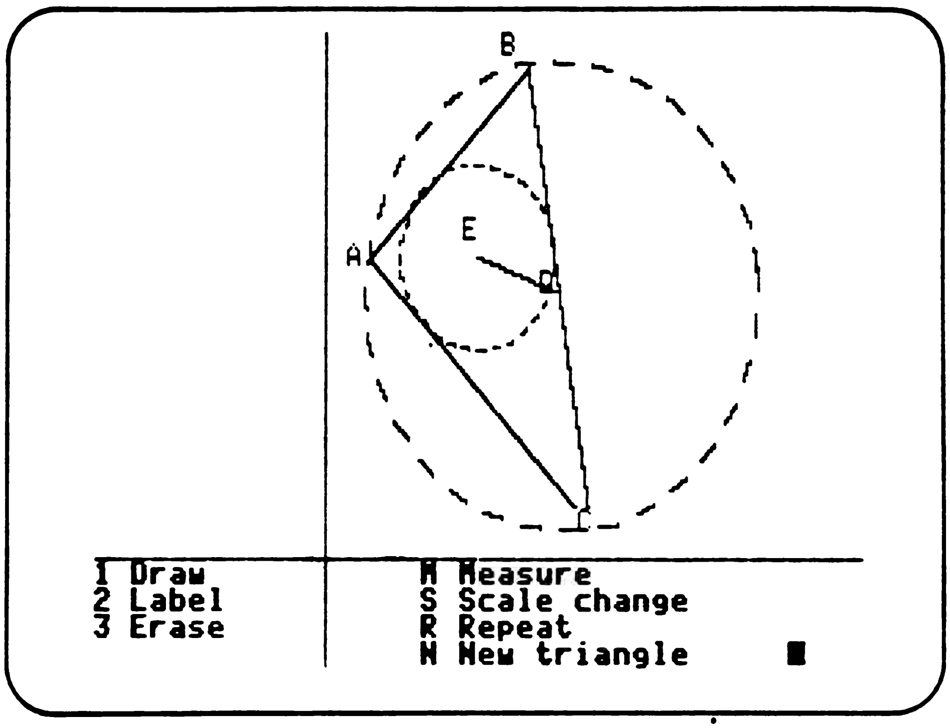
Repeat construction:
1 on new triangle
2 on previous triangle

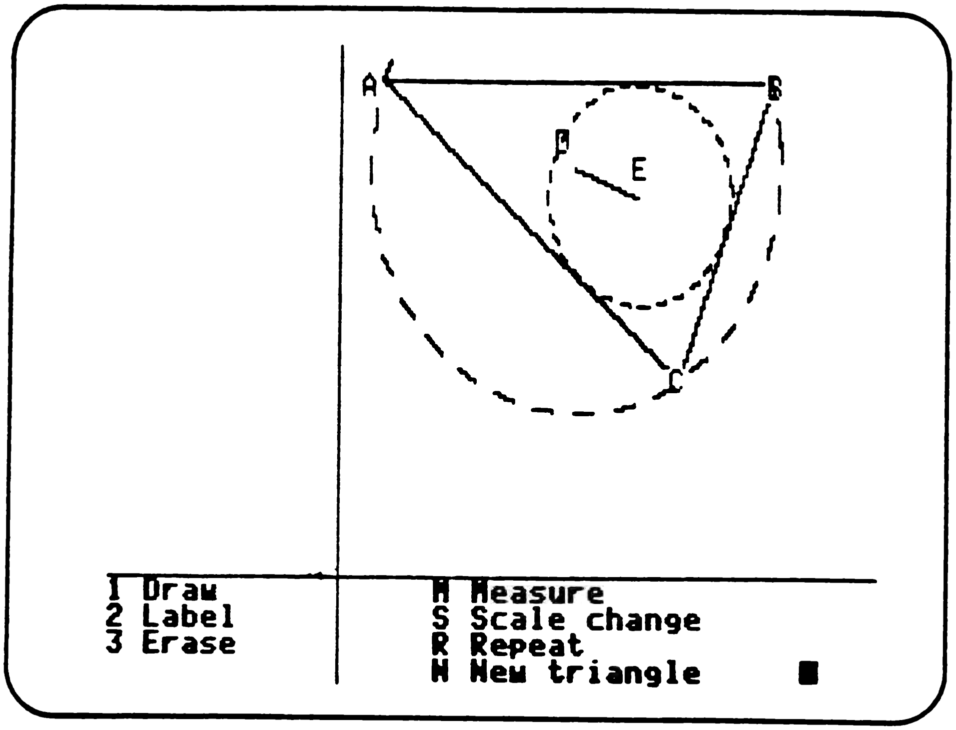
Option 1, on new triangle, enables you to see the the construction(s) you have made carried out on a new and different triangle. After entering 1, the SUPPOSER will return to the triangle selection menu (described in detail in Section II.2 above) and ask you to specify the kind of triangle on which you wish to see the construction(s) repeated. Now enter the number of the triangle on which you want the construction(s) carried out. The new triangle will be drawn and the message will now read:

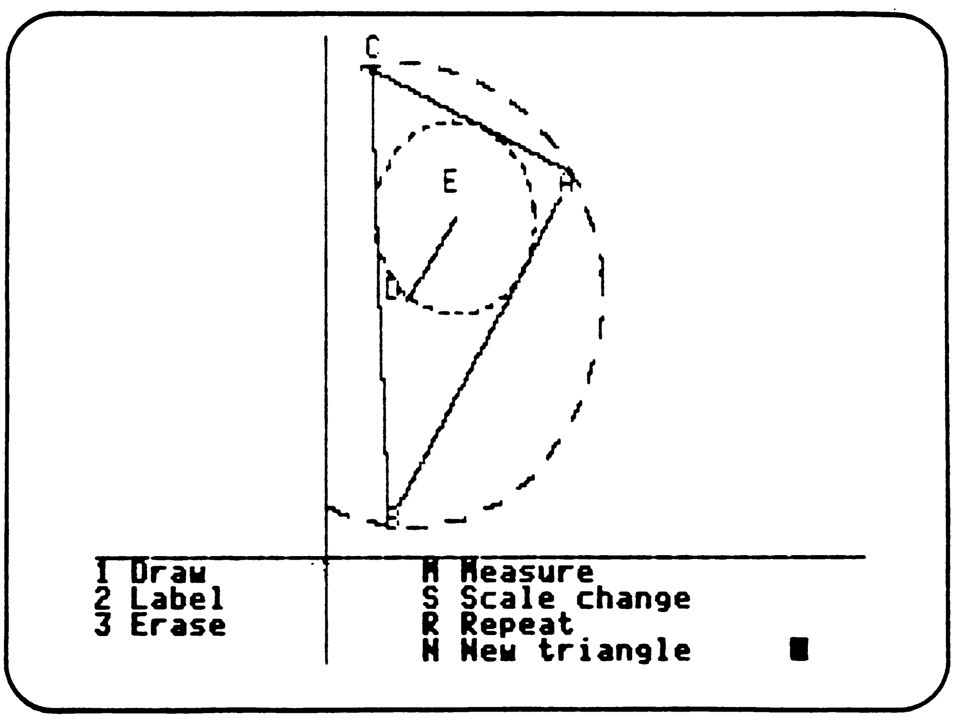
Press SPACE BAR to repeat construction.

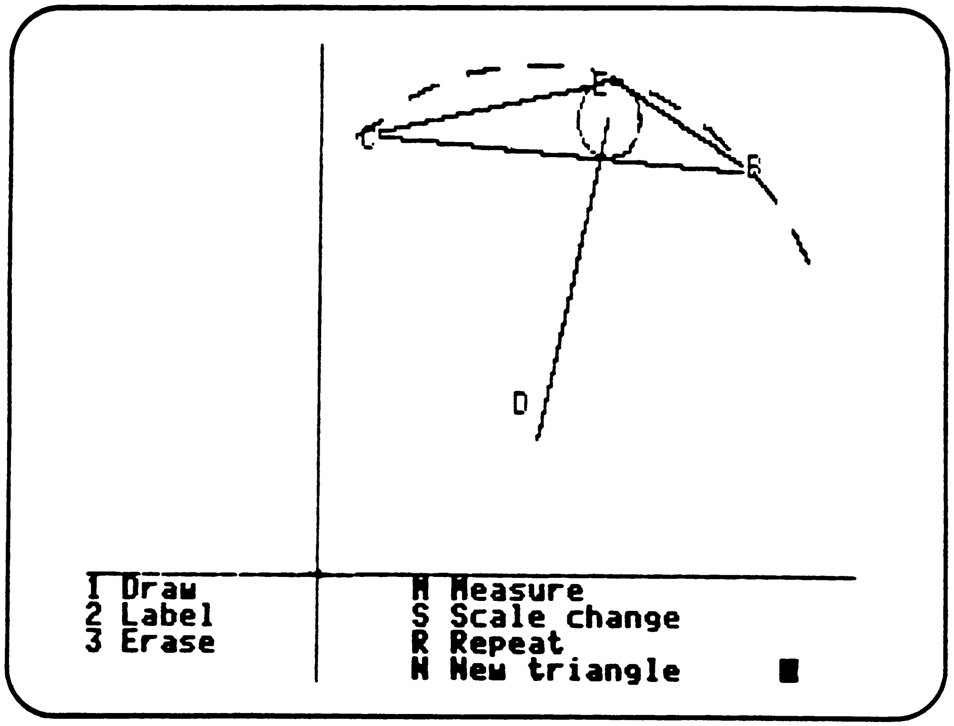
Each time you press the SPACE BAR, a single step in the construction will be repeated. Continue to press the SPACE BAR until all steps in the construction have been replicated. When all the steps in the construction have been carried out, the SUPPOSER will return you to the MAIN MENU.

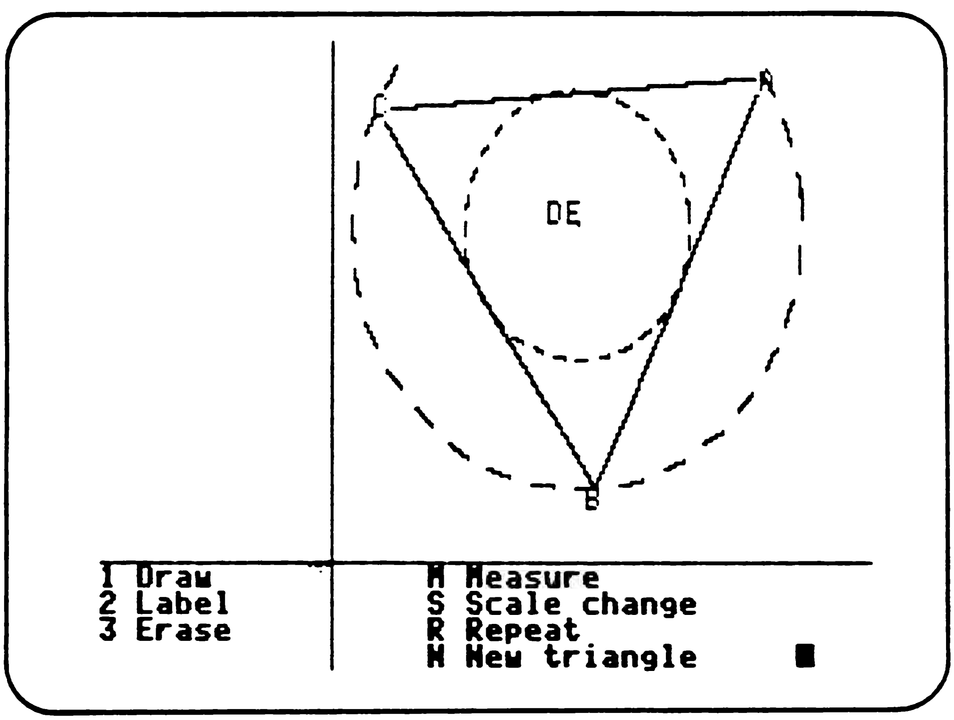
Here, for example, is a construction which joins the centers of the inscribed and circumscribed circles of triangle ABC. The construction is shown for an acute, a right, an obtuse, an isosceles and an equilateral triangle.




A
B
D
E
C
1 Draw
2 Label
3 Erase
M Measure
S Scale change
R Repeat
N New triangle


C
E
A
D
1 Draw
2 Label
3 Erase
M Measure
S Scale change
R Repeat
N New triangle


C
E
B
D
1 Draw
2 Label
3 Erase
M Measure
S Scale change
R Repeat
N New triangle


C
A
DE
B
1 Draw
2 Label
3 Erase
M Measure
S Scale change
R Repeat
N New triangle

Option 2, on previous triangle, enables you to see the construction(s) you have made carried out on a triangle that you worked with earlier.

After entering the number 2, the message will read:

<== ==> to see triangle:
1 2 3 4
Press RETURN to select

The program stores the four most recently constructed triangles and you can use the arrows keys to move back and forth among the four. As you move from number to number, each triangle will appear on the screen and its corresponding number will be highlighted. When you have identified the triangle on which you would like to see your construction(s) repeated, press RETURN. The message will now read:

Press SPACE BAR to repeat construction.

Each time you press the SPACE BAR, a single step in the construction will be repeated. Continue to press the SPACE BAR until all the steps in the construction have been replicated. When all the steps in the construction have been carried out, the SUPPOSER will return you to the MAIN MENU.

IMPORTANT NOTE: If you have used the adjustable element(s) option in MEASURE, the SUPPOSER will not repeat those parts of the construction connected to the movable point(s) (labeled with inverse letters) introduced into the construction. In addition, the SUPPOSER will not repeat any part of the construction made after the first use of the adjustable element(s) in the construction.

New Triangle

The New Triangle option allows you to select a randomly generated triangle or to create your own triangle on which to carry out constructions.

To select this option, enter the letter N. The screen will clear and the message will read:

1 Right	4 Isosceles
2 Acute	5 Equilateral
3 Obtuse	6 Your own

Choices 1 through 5 generate a random right, acute, obtuse, isosceles, or equilateral triangle, respectively. Choice 6 allows you to define a specific triangle of your own making on which to make your construction.

For a detailed description on the operation of this option, see Section I.2, How Do I Make A Triangle?

III. USING THE SUPPOSER AS A TEACHING TOOL

This section describes some of the ways in which THE GEOMETRIC SUPPOSER can be used with students. To set the tone for that discussion, here are some entries from the diary of Richard Houde, a mathematics teacher in the Weston (MA) High School who used THE GEOMETRIC SUPPOSER with his students during the 1983-84 school year.

"...What a surprise! Triangles of all shapes and sizes could be displayed on a screen in a matter of seconds; measures of angles, sides, perimeters and areas could be obtained instantaneously; and line segments could be drawn in triangles and then redrawn and repeated in other triangles using just a few key strokes. But best of all, students found that they could "suppose" relationships existed, for example, between line segments in triangles or areas of sections in triangles and then could check in a matter of minutes whether or not these relationships were true. This year I made the SUPPOSER a formal part of my two semester geometry classes; so much in fact, that I did not distribute geometry textbooks to my students. The SUPPOSER, worksheets and I became their textbook. I wrote problems for the SUPPOSER that asked students to state conjectures about geometric relationships. These conjectures actually represented many of the theorems one might find in a traditional high school geometry textbook. I also wrote problems for the SUPPOSER that extended simple geometric ideas to more complex ones and again asked students to state conjectures.

...I believe that the problems I posed to students required them to think. Students had to: (1) understand the statement of a given problem before they could begin to investigate it, (2) collect and organize data, (3) develop and check procedures for analyzing data, (4) exhibit counterexamples to show that conjectures were false, (5) write proofs for conjectures that they believed were true, and (6) organize and prepare written reports.

...Students worked independently on these problems both during and outside class time and occasionally asked me for hints or to clarify problem statements. When students found that they could not draw any conclusions regarding a given problem, they would group together to discuss it. These conversations often resulted in a restatement of the problem and the students went back to work on it again. When a group of students met to share their results, it was also not uncommon for them to argue whether or not their conjectures were true or false. In fact, I found some of the best problems for the course while listening to these conversations or trying to answer their questions in class. These times became the most exciting in the course for me because we were all creating our own mathematics together. During one particular class, when I mentioned that perhaps

it was about time to start distributing a textbook, one student responded, `It's much more fun this way (without a textbook) because we are coming up with geometry ourselves.' Surprisingly, to me at least, the rest of the students nodded their heads in agreement."

Using THE GEOMETRIC SUPPOSER as part of a geometry class transforms learning geometry into making geometry. With this transformation, the balance shifts from proof to conjecture, from solutions to questions, from seeking answers to encouraging inquiry and investigation. For students, this means greater responsibility since the course of learning relies heavily on the findings of students and on the reactions of their classmates. For teachers, this may require some flexibility and patience in their efforts to provoke and to guide productive exploration.

The key to effective classwork with THE GEOMETRIC SUPPOSER is discussion. To create an environment conducive to rich and lively discussion several conditions need to be met:

-- There should be an interesting subject to discuss.

-- In order to participate and to contribute, every student should have knowledge of the subject under discussion gained from direct experience exploring the subject either individually or in a group.

-- The atmosphere in the classroom should be one that welcomes discussion of any plausible idea, and one in which every student believes that such ideas are proper subjects for class discussion.

EXAMPLES OF CLASSES USING THE GEOMETRIC SUPPOSER

Here are two examples of classroom use of the SUPPOSER. The first is a class with a single computer; the second is a class following a computer lab session with the SUPPOSER.

Class A: Using the SUPPOSER to Introduce or to Evoke New Ideas

There is one computer in the classroom. It is helpful to have a large monitor in the front of the room or several small monitors around the room so that everyone in the class can see the display. (It is assumed students do not have individual access to computers on a regular basis and the SUPPOSER is being used here as a tool to help the teacher evoke ideas, to answer questions, and to provide the class as a whole with an opportunity to explore a variety of notions.)

Suppose the class knows that a triangle can be circumscribed with a circle. It is, of course, possible at this stage to teach the formal proof that the intersection of the perpendicular bisectors is the center of the circumscribed circle and show that there is only one such point and one such circle. What follows is a list of ideas generated by students about this problem. To be sure, different students were interested in different entries on the list and the degree of understanding varied from student to student.

-- For a given triangle, is there only one circumscribing circle?

-- Is the center of this circle inside the triangle, outside the triangle, or on the triangle?

-- Why is the center of the circumscribed circle where it is?

-- Can you locate the center of the circumscribed circle exactly?

-- Is there any relationship between the size of the circumscribed circle and the size or type of the triangle?

-- Is there anything in common among all the triangles that can be inscribed in the same circle?

The teacher produces several examples of circumscribed triangles using the SUPPOSER and asks the students to say what they can about the location of the center. Here is the discussion that follows:

-- Can you draw some more examples?

-- Can I try another case?

-- I want to look at another right triangle.

The student uses the SUPPOSER to try another right triangle.

The class comes to a conclusion about the center in right triangles.

-- Is that always true?

-- We have tried four triangles!

The teacher indicates that the class will have to devise a proof at a later time in order to be certain.

-- May I try an equilateral triangle? I want to draw a circle which has its center where the medians intersect and its radius is the segment from the center to one vertex.

-- Will that be just the same with another triangle?

This discussion lays the foundation for devising hypotheses about the centers of circumscribed triangles and for arriving at the conclusion that these hypotheses are in no way dependent on the size of the triangles.

Where students do not have individual access to the computer, problems should be used that provide the class with a collective challenge and the discussion should be managed so that students see themselves as part of a team trying to solve a puzzle.

Such discussions can occur at any time, but are more likely to be productive when a new topic or subject is introduced. Use the SUPPOSER to introduce the new geometric phenomena and to stimulate the investigation.

Class B: Using the SUPPOSER to Examine Data and to Make Conjectures

When students have access to a computer outside of class time, a key role for the teacher during class time is to help students analyze their data and findings. The teacher becomes a partner in the conversation, helping students focus their arguments and look for evidence in their data. The process of students reporting to their classmates can be, and often is, an occasion for sharply differing opinions about some geometric phenomenon.

Here is an example drawn from a class discussion. It is the first quarter of the year in an average level class. The students had been given an assignment that included the problem:

> Consider a random point inside a triangle. Find a relationship between the sum of the three segments joining that point to the vertices of the triangle and the perimeter of the triangle.

The teacher picks one set of papers from among those turned in and puts up the following table:

AD+BD+CD	Perimeter
9.8	17.9
11.6	18.0
8.1	14.3
12.1	21.0

-- I did it. I tried to do lots of things and could not find any connection. I tried to subtract and look for ratios but it did not come out.

-- I also subtracted and found that in the triangles that I

tried the difference between the sum and the perimeter was between 6 and 7.' I can see now on her table that that is not always true.

-- I have found that the sum is always smaller than the perimeter.

Class: That is not enough! That is obvious! (Several students say that they saw that but that they did not think it worth reporting.)

Teacher: You have such a nice table. I want you to take a guess now. Suppose you have a triangle with 30 for its perimeter, what will be the sum of the 3 segments?

-- It will be 17.

Teacher: Why?

-- Because I know that it should be a little bit more than half of the perimeter. I did not write that because I did not want to write something which is "about the size" and not "exactly". (This student had clearly discovered and understood that the sum of the three segments must exceed half the perimeter.)

Students reporting back to the class about the results of their inquiries in the lab provides a wonderful opportunity for teachers to gauge the level and degree of students' understanding. (In the episode above, we can see the level of students' understanding about inequality and precision.) Further, it is a fine source for suggestions and ideas for further class exploration and discussion. As exciting as such discussion may be, there will be a point at which the teacher may wish to stop the general discussion and ask the students to write their conclusions or to devise proofs of what they believe.

Discussions of this sort are likely to be richer in content and vigor after the students have had an opportunity to explore some issue thoroughly. On the other hand, there can be brief discussions resolving the uncertainties of what some students call "strange drawings."

GETTING STUDENTS STARTED AND WORKING PRODUCTIVELY

To help students learn how to use THE GEOMETRIC SUPPOSER, demonstrate the use of the program to them with some simple problems drawn from your course materials. Here are two examples of simple problems that will help students learn about the SUPPOSER:

Problem: What is the definition of an isosceles triangle? What are its properties?

With the SUPPOSER:
Draw an isosceles triangle.
Measure the angles and sides.
Scale it, measure again.
Draw a median.
Draw the remaining medians.
Measure all six angles at the vertices.
Repeat on another isosceles triangle.

Problem: What are two ways of replicating a triangle?

With the SUPPOSER:
1. Create a parallelogram starting with any triangle.
2. Use the Reflection option

From students and teachers, we've gathered some helpful hints about how to minimize discomfort and maximize learning in using the SUPPOSER:

-- To help students overcome the common fears of making mistakes or losing information, emphasize the following:

You can always cancel your most recent key press by pressing ESC.

At any time, the program remembers the four most recent triangles on which you have been working and can repeat your construction on any of them.

Keep a written record of what you've done. Include such things as the steps you took to make a particular construction as well as all the measurements you made on that drawing.

-- In order to help the students record drawings, measurements and procedures, the teacher shoud give particular attention to the layout of the written assignment sheets. Make liberal use of tables to be filled in and blank space for drawings to be recorded.

-- Teach the use of the Repeat option in the SUPPOSER as early as possible. In addition to offering the students an opportunity to explore their ideas and conjectures, the Repeat option allows the students to remake constructions with greater ease and efficiency.

-- Suggest that students adopt the following techniques and strategies in using the SUPPOSER to explore problems:

Look at many examples;

Take notes;

Look at the data, make guesses that may be suggested by your data, and explore them in other cases;

Share ideas, difficulties, and interesting findings with your classmates or your partners in the lab.

WORKING WITH THE GEOMETRIC SUPPOSER IN A COMPUTER LAB

Where a geometry class has access to a computer lab, students can work in the lab during class time, in their free time during the day, or before and after school. For this kind of use of the SUPPOSER, the teacher should prepare assignments or problem sets in writing for students to work on.

We have seen teachers use lab time effectively in two different ways:

- -- having students work on written problems in the lab on their own for two or three hours per week, or,
- -- holding class in the lab one or two times per week and using the remaining class time to introduce new topics and discuss findings from the lab.

Students should always be asked to explain for themselves and to the class why they chose to explore a particular direction or make a construction on a particular triangle. Can the data be manipulated to find a pattern? When a student first engages in the subject, these arguments may be informal. As they become both more comfortable and more nimble with the subject, it is to be expected that these arguments will take the form of a formal written proof.

Lab work with the SUPPOSER is most successful when students work in pairs. With a sufficient number of computers, students can work individually, but because of the open-ended nature of the inquiry, the educational experience is likely to be richer when students can put their heads together and are forced to subject their ideas and actions to another's scrutiny. In cases where there are too few computers for students to work in pairs, we advise finding a way to schedule smaller groups of students.

PREPARING PROBLEMS AND WRITTEN MATERIALS FOR USE WITH THE GEOMETRIC SUPPOSER

With the SUPPOSER, students can draw, study, and solve most geometry problems in a traditional high-school geometry text book. But this approach doesn't really exploit the instructional potential of the SUPPOSER because problems in traditional texts tend not to be open-ended, and are not well-suited to exploration. It is however often possible to start with problems presented in a traditional text and to change the statement of the problem so that it becomes an open-ended question that calls for research and investigation. In working with teachers, we've seen them create three different types of problems to use with the SUPPOSER at different points in the curriculum and with students of varying ability. Here are examples of the three types of problems and how they relate to problems in a traditional text:

Defining and Exploring a New Concept

In the textbook: Often in a chapter review or problem set, there will be items such as: State the perpendicular bisector theorem.

With the SUPPOSER: Find the properties of a perpendicular bisector, using the Repeat and Measure options. Collect data to support your conjectures.

OR

Instead of giving the class a definition, try sending them to the lab with tasks such as these:

Find out what an altitude is.

Report back to the class as many properties as you can find about angle bisectors.

In each case, ask students to gather evidence to support their arguments.

Establishing True or False Statements

In the textbook: Prove the exterior angle theorem (An exterior angle of a triangle is greater than each of its remote interior angles.)

With the SUPPOSER: Each of the following statements is true or false. In each case state whether you think it is true or false and why you think so.

-- An exterior angle is always greater than any interior angle of a triangle.

-- An exterior angle is always an obtuse angle.

-- The sum of the exterior angles and at least one of the interior angles is 180 degrees.

This kind of problem can introduce students to the fact that there are false statements and that gathering evidence and looking for counterexamples is essential before reaching conclusions.

Exploring Open-Ended Problems

In the textbook: Given triangle ABC, where AB=AC and where D and E are points on BC and BD=DE=EC, prove that triangle ADE is an isosceles triangle.

With the SUPPOSER: Make a triangle and subdivide one side.
State as many conjectures as you can about your drawing.
Check your conjectures to see if they hold for other types of triangles, e.g., isosceles, equilateral, obtuse.
Prove at least one of your conjectures.

OR

Pose a problem which opens up a wide area of inquiry.

What are the properties of a median?

Does a median bisect the side to which it is drawn? Does it bisect anything else?

Can you find the properties of a median in particular triangles which are not true for all triangles?

Do medians drawn from different vertices have the same or different properties?

These are only suggestions. Obviously, as you work with your students and the SUPPOSER you will be able to devise challenges and problems suited to their needs and interests. (For examples of additional problems and worksheets, see the next section of the manual.)

IV. EXAMPLES AND SUGGESTED EXERCISES

Most of the exercises and problems that follow involve making conjectures. Many students have difficulty making conjectures. Further, many students have difficulty with notions of plausibility and proof. Here are some suggestions for introducing these issues.

To begin with, it is suggested that you discuss the following propositions with your students.

> If you believe a conjecture to be FALSE you must offer a counterexample in order to disprove it.
>
> In order to prove a conjecture FALSE, it is <u>sufficient</u> to offer a single counterexample.
>
> If you believe a conjecture to be TRUE you should be able to offer a convincing argument to support your belief.
>
> In order to prove a conjecture TRUE, it is <u>not sufficient</u> to offer examples of cases that are consistent with the conjecture.
>
> Why is it sufficient to offer a single counterexample in order to disprove a conjecture, and yet no number of confirming examples is sufficient to prove a conjecture?

The following section contains a series of activities for students. Some of them are quite simple and may be worked through in the space of part of a class period or as part of a regular homework assignment. Others are more elaborate and should be thought of as a sort of mini-research project. Depending on the available time and the level of your students, you may want to break up exercises with several parts into smaller pieces.

Find the measures of the interior angles of a triangle. What do you believe to be true about each of the angles? about the sum of any two? about the sum of all three?

Here is a chart you may find useful.

ANGLE MEASURES AND SUMS

Type of Triangle	ABC	BCA	CAB	ABC + BCA	BCA + CAB	CAB + ABC	ABC+BCA+CAB

Conjectures: 1.

2.

3.

Draw a triangle ABC. Extend side AB from point A to a point D. How is angle DAC related to angle BAC? to angles ABC and ACB?

State a conjecture that describes a relationship between an exterior angle of a triangle and one, two or three of its interior angles.

The following problems present different ways of exploring the notion of median.

State a conjecture about the medians of any triangle.

State a conjecture about the lengths of the line segments formed by the intersection of medians.

In triangle ABC draw a median to side BC from vertex A. Label the intersection D. State at least two conjectures about the relationship between triangles ABD and ACD.

Is it true that a median drawn from a vertex bisects the angle at the vertex from which it is drawn?

Is it true that a median bisects the perimeter of a triangle?

Is it true that a median bisects the area of a triangle?

Is it true that a median bisects the side of the triangle to which it is drawn?

For each question, inspect a number of cases and record your data. From your data make a conjecture.

Draw a median from the right angle of a right triangle to the hypotenuse of the triangle. State a conjecture about the relationship between the length of the median and the length of the hypotenuse. Find the class of triangles for which your conjecture is true.

Is it true that all isosceles triangles are acute?

The SUPPOSER always draws isosceles triangles with sides AB and AC equal. Here is a chart you may find useful.

Triangle	Angle BAC	Angle ACB	Angle CBA

Is it true that in a right triangle, the measure of each of the non-right angles is less than 90 degrees?

Draw all the altitudes of an acute triangle. State a conjecture about the three altitudes of an acute triangle. Is your conjecture true for an obtuse triangle?

Find at least three different ways to compute the area of any triangle.

Using the measure option, state at least four conjectures about triangles and midsegments. (You may find it useful to draw all the midsegments of a triangle and then take some measurements.)

Place a point D at random in triangle ABC. Draw segments DA, DB and DC. Compare the sum of the lengths of these segments with the perimeter of the triangle. Make a conjecture and test it on a number of cases.

Use the following chart.

	Lengths				
	DA	DB	DC	DA+DB+DC	Perimeter
a.					
b.					
c.					
d.					
e.					
f.					

Conjectures: 1.

2.

3.

Given a triangle ABC. Can you locate the center of the inscribed circle? What is the radius of the inscribed circle? Draw the circle and check to see that your procedure works with other triangles as well.

Given a triangle ABC. Can you locate the center of the circumscribed circle? What is the radius of the circumscribed circle? Draw the circle and check to see that your procedure works with other triangles as well.

A challenging problem. Draw a triangle ABC. Extend side AB from B, and extend side AC from C. Now draw a circle that is tangent to segments DB, BC and CE. Repeat your construction with other triangles.

WORKING WITH THE APPLE II+

(1) Turn on the television or monitor.

(2) Insert the diskette into the disk drive with the label facing up and on the right.

(3) Close the door to the disk drive.

(4) Turn on the Apple.

(5) You will see a red light on the disk drive turn on. If the disk drive light does not turn off in about 10 seconds, turn the Apple off and make sure your diskette is placed correctly in the drive.

(6) SUNBURST will appear on the screen, followed by the program name.

(7) Follow the directions in the program.

(8) If you wish to stop during the program, hold down the Control key and press E.

Shutting Off the System

(1) Remove the diskette from the disk drive and return it to its place of storage.

(2) Turn off the Apple.

(3) Turn off the television or monitor.

Apple IIGS: Control Panel Settings

To allow your Apple IIGS to work properly with Sunburst software, certain Control Panel settings should be selected. The Apple IIGS retains these settings even after the power is turned off.

To Use the Control Panel:

- Turn on the Apple IIGS and monitor.
- Enter the Control Panel main menu by holding down the CONTROL and OPTION keys, and then press RESET (the rectangular key located above the number keys). If your Apple IIGS is in an Apple //e case, use the closed-apple () key instead of OPTION.
- Press the 1 key to enter the Control Panel.
- Use ↓ and ↑ to highlight the feature you want to change and press RETURN. Again use ↓ and ↑ to highlight a specific option and change it by using the ← and → keys.
- After you have finished making changes, select Quit to use the Apple IIGS.

To Change the Display:

- Highlight **Display** and press RETURN.
- Set **Type** to **Color**.
- Set **Columns** to **40**.
- Set **Text** to **White**.
- Set **Background** to **Black**.
- Set **Border** to **Black**.
- Press RETURN to save the changes and to go back to the Control Panel.

To Change the System Speed:

- Highlight **System Speed** and press RETURN.
- Set **System Speed** to **Normal**.
- Press RETURN to go back to the Control Panel.

To Change the Slots:

- Highlight **Slots** and press RETURN.
- Set **Slot 1** to **Printer Port**. If you are using a printer card, select the slot number your printer card is in.
- Set **Slot 6** to **Disk Port**, if you use a 5.25 - inch drive connected to the disk drive port.
- Set **Slot 6** to **Your Card**, if you use a 5.25 - inch drive connected to a controller card in Slot 6.
- Set **Startup Slot** to **Scan**.
- Press RETURN to go back to the Control Panel.

WHAT HAPPENS IF...?--SUNBURST COURSEWARE AND WARRANTY

1. What happens if a program will not load or run?
   Call us on our toll-free number and we will send you a new diskette.

2. What it I find an error in the program?
   We have thoroughly tested the programs that SUNBURST carries so we hope this does not happen. But if you find an error, please note what you did before the error occurred. Also, if a message appears on the screen, please write the message down. Then fill out the evaluation form or call us with the information. We will correct the error and send you a new diskette.

3. What happens if the courseware is accidentally destroyed?
   SUNBURST has a lifetime guarantee on its courseware. Send us the product that was damaged and we will send you a new one.

4. Can I copy this diskette?
   The material on the diskette is copyrighted. You <u>should</u> <u>not</u> copy the courseware.

5. Can I take this diskette out of the computer after the program has loaded and put it into another computer?
   Yes!